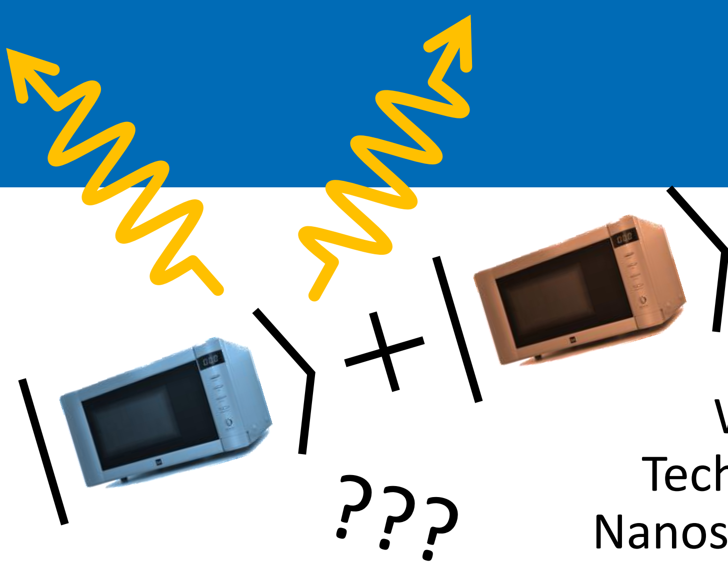
Bayerische Akademie der Wissenschaften

Technische Universität München

Measurements using squeezed microwaves

Continuous-variable propagating quantum microwaves

September 25, 2018

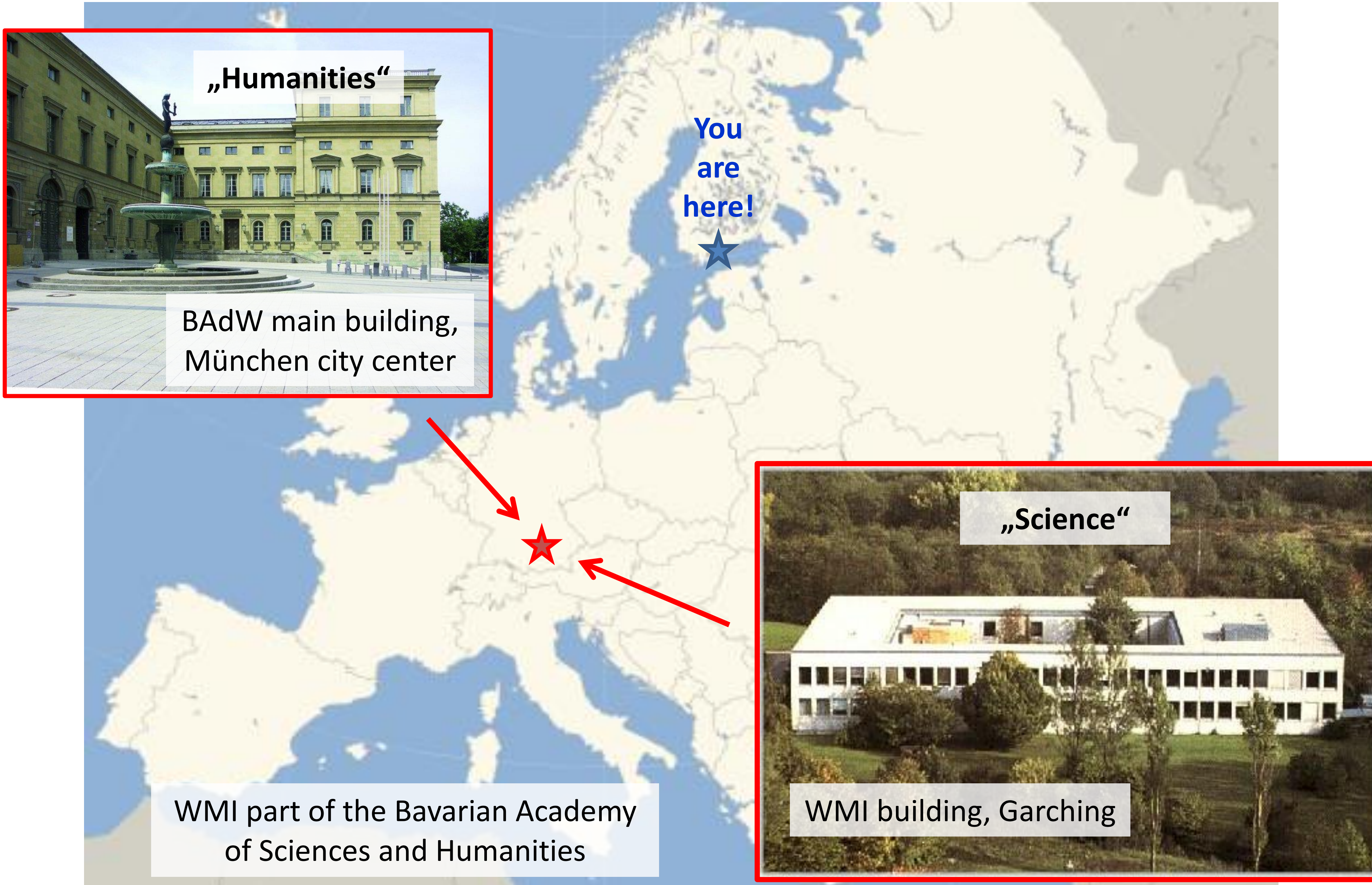


Frank Deppe

Walther-Meissner-Institut
Technische Universität München
Nanosystems Initiative Munich (NIM)

MUNICH QUANTUM CENTER

Walther-Meißner-Institut



Microwaves

Microwaves are omnipresent in everyday life!

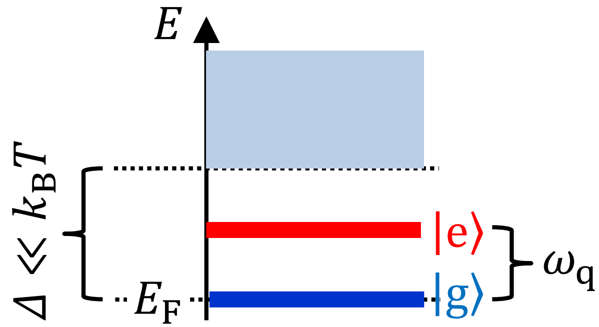
Described well by classical electrodynamics → „Classical“

Quantum effects (superposition, entanglement)?

→ Need platform for generation, manipulation & measurement!

Superconducting quantum circuits

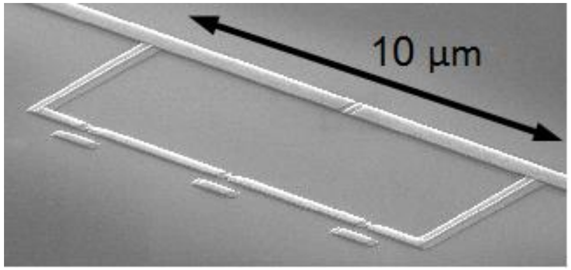
Superconductivity



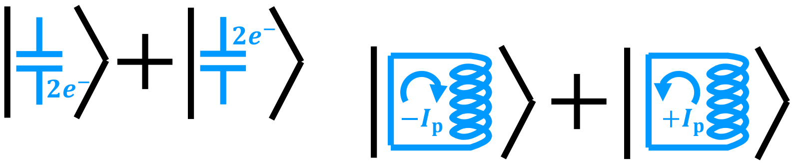
Superconducting quantum circuits

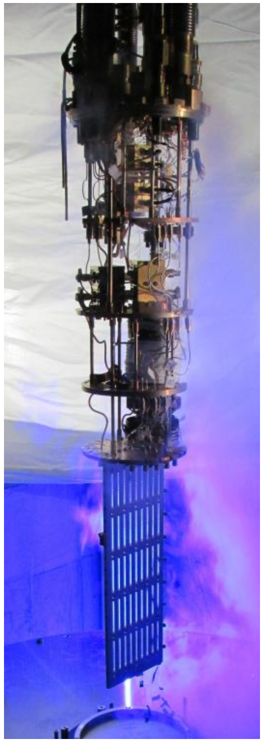
Aluminum

$\Delta/h \simeq 50$ GHz



Superpositon & entanglement





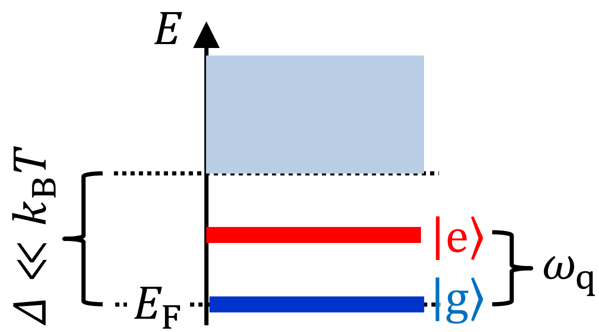
Millikelvin temperatures

1GHz ⇔ 50 mK

→ **Quantum computing, simulation, communication, illumination, sensing**

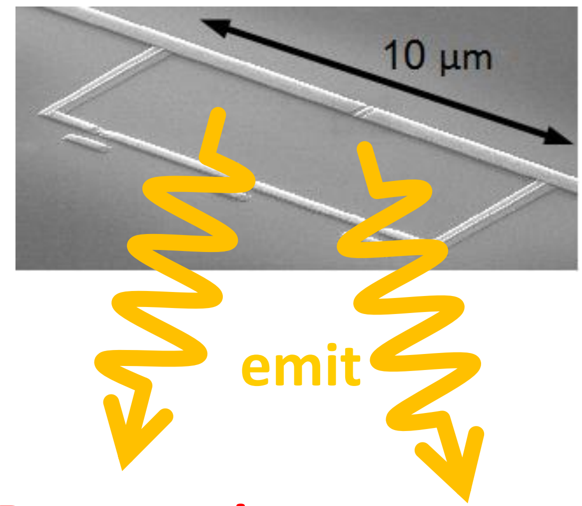
Superconducting quantum circuits

Superconductivity

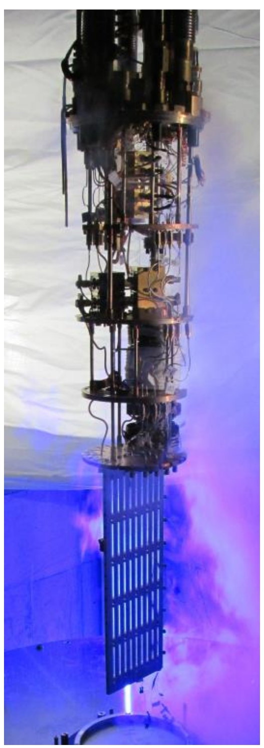


Superconducting quantum circuits

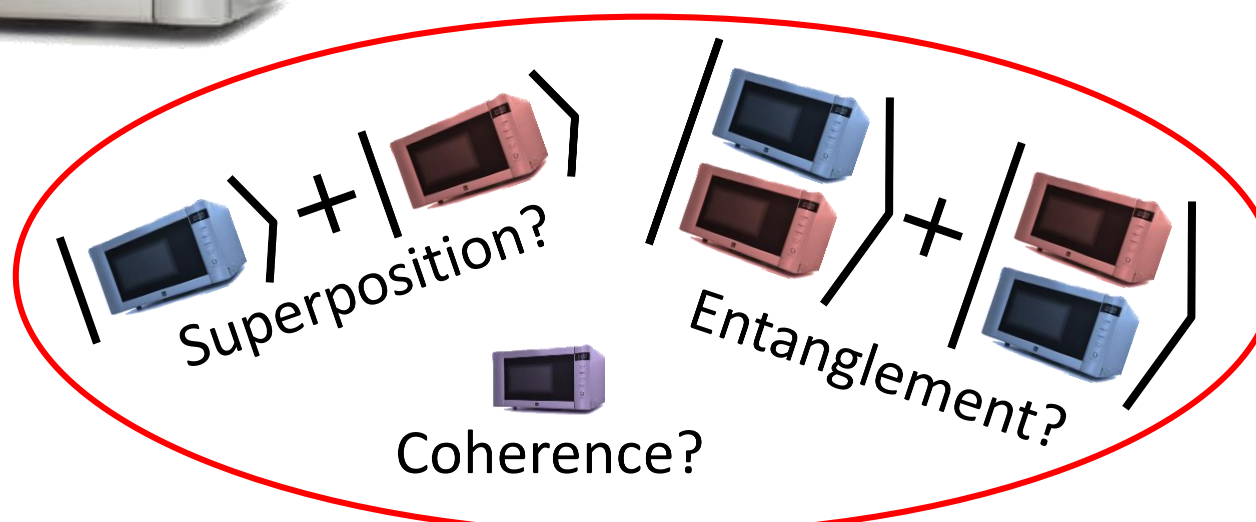
Aluminum
$\Delta/h \simeq 50$ GHz



Propagating quantum microwaves

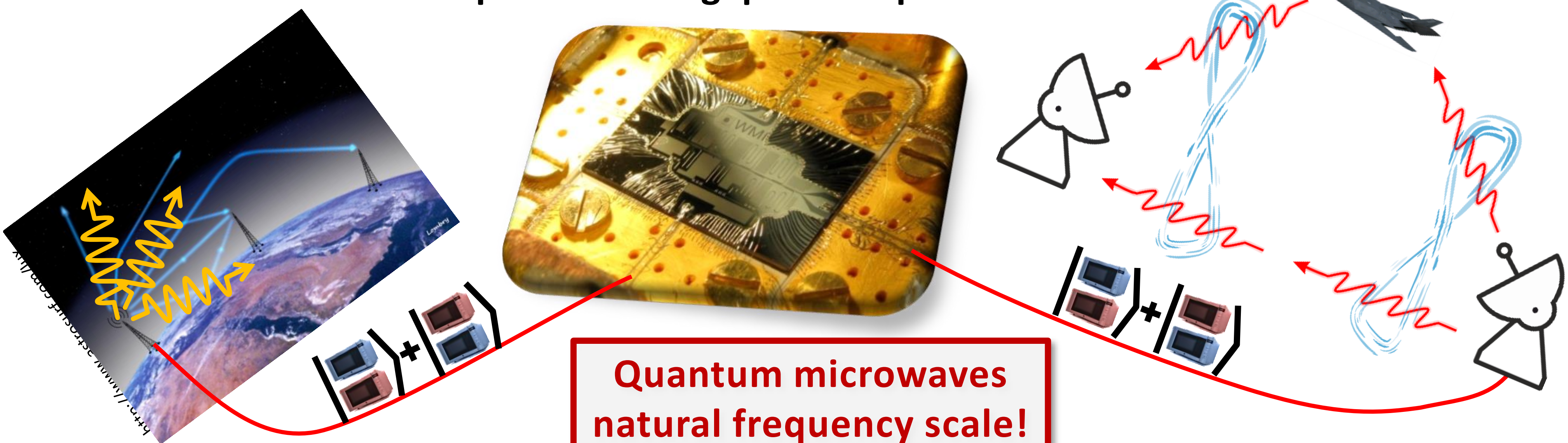


Millikelvin temperatures
$1\text{GHz} \Leftrightarrow 50$ mK



Quantum $|\text{Science}\rangle + |\text{Technology}\rangle$

Superconducting quantum processors



Quantum microwaves natural frequency scale!

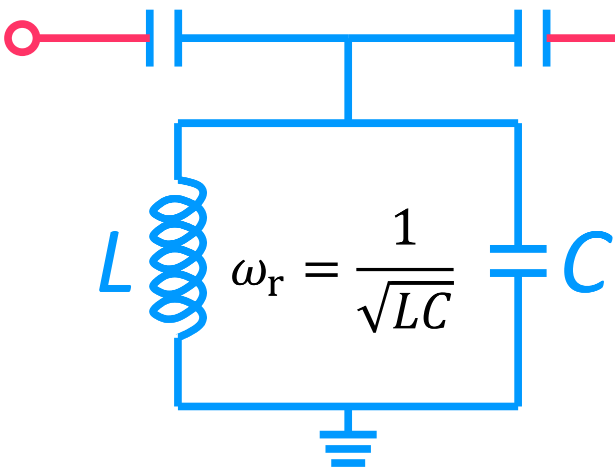
- ☺ No frequency-conversion losses
- ☺ Low-loss superconducting waveguides
- ☹ Low photon energy

Quantum applications

→ Quantum communication (teleportation)
→ Distributed quantum computing (quantum LAN)
→ Quantum illumination (quantum radar)

Quantum harmonic oscillator

LC Resonator → Box for quantum microwaves



$$E_L = \frac{\Phi^2}{2L}$$

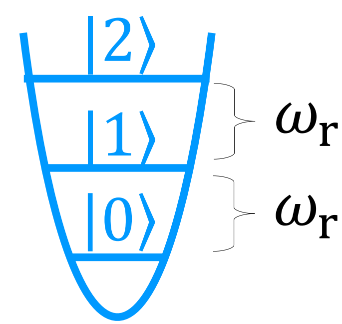
$$E_C = \frac{Q^2}{2C}$$

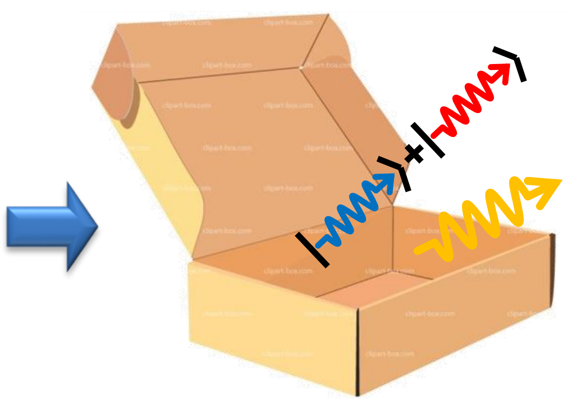
$\hbar\omega_\mathrm{r} > k_B T$

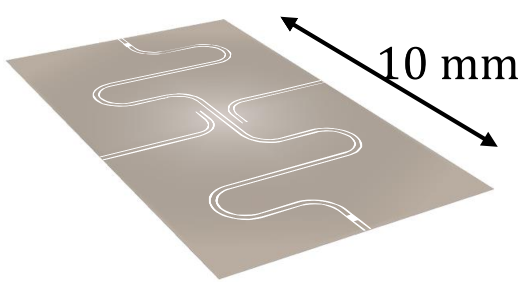
$\Phi \rightarrow \hat{\Phi}$, Q $\rightarrow \hat{Q}$

Ladder operators $\hat{a}^\dagger$, $\hat{a}$

Eigenstates → Photon number states $|n\rangle$



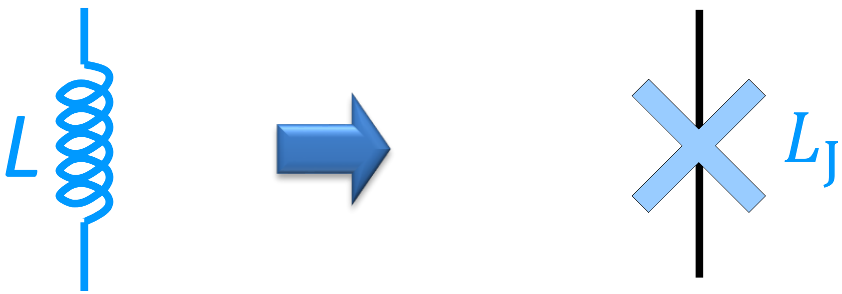




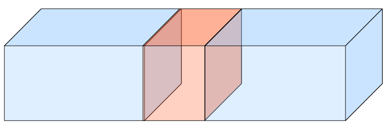
$$\hat{H} = \hbar\omega_\mathrm{r}\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$$

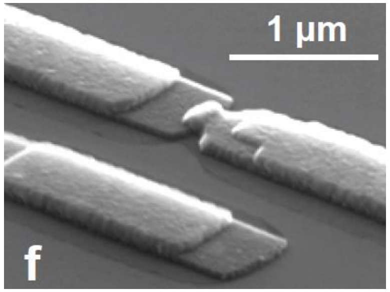
$[\hat{\Phi}, \hat{Q}] = i\hbar \Leftrightarrow [\hat{a}, \hat{a}^\dagger] = 1$ → **„Quantum 2.0“**

Nonlinear quantum circuits



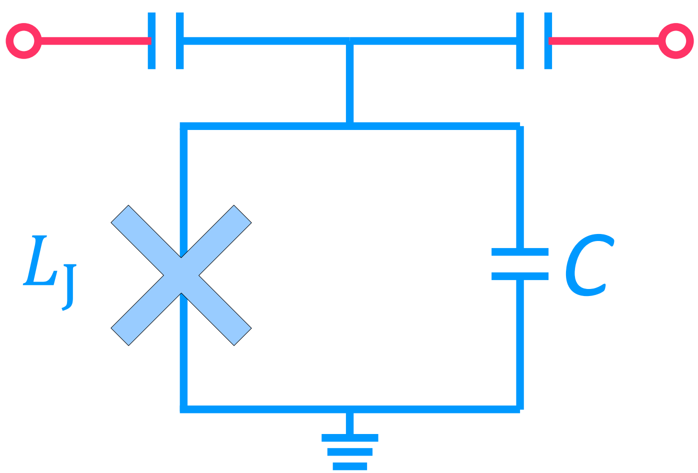
SIS Josephson junction



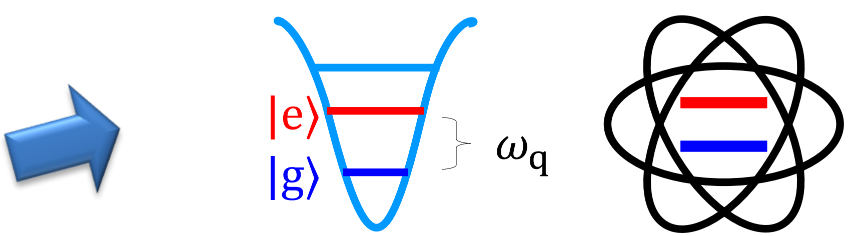


Josephson inductance L_J

Nonlinear, can be negative, flux-tunable in a small loop



Anharmonic oscillator



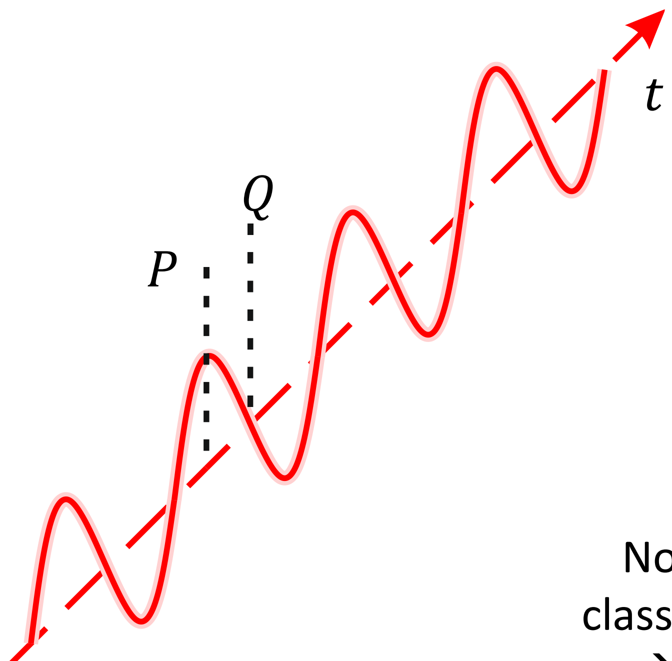
Qubit

Quantum swing (JPA)

Continuous variables

Discrete variables → Photon number states $|n\rangle$ (discerete basis)
→ Particle picture

Wave-particle duality → Wave picture?
→ Continuous variables!

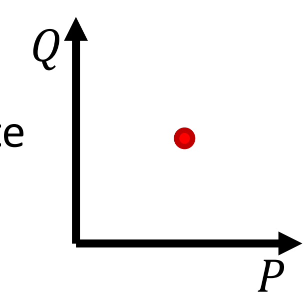


$$A\cos(\omega t + \varphi) = Q\cos(\omega t) + P\sin(\omega t)$$

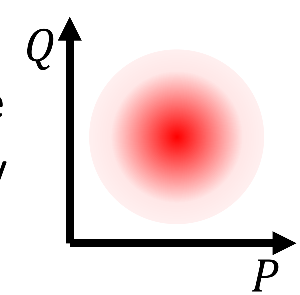
A, φ inconvenient → Field quadratures Q, P

Phase space representations

Noiseless classical state → Point



Noisy classical state → Probability distribution



Continuous variable quantum states

Quantization → $\hat{P}$, $\hat{Q}$ pair of conjugate variables analoguous to $\hat{x}, \hat{p}$

$$[\hat{P}, \hat{Q}] = i \quad \Leftrightarrow \quad (\Delta P)(\Delta Q) \geq \frac{1}{4}$$

Phase space representation → Quasiprobability distribution („Wigner function“)

$$W(Q,P) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left\langle Q - \frac{\zeta}{2} \middle| \hat{\rho} \middle| Q + \frac{\zeta}{2} \right\rangle e^{ip\zeta} d\zeta$$

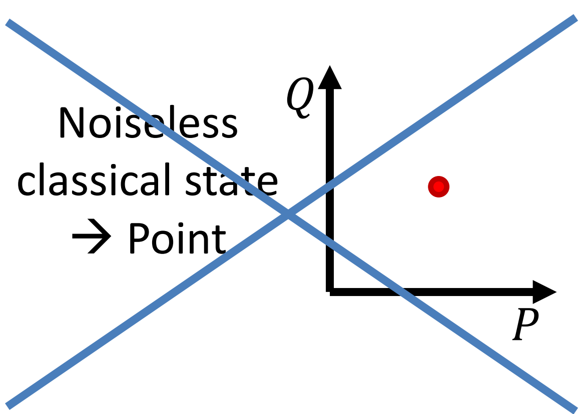
→ Equivalent to

→ Denisty matrix $\hat{\rho}$

→ Characteristic function (moments)

→ $W(Q,P)$ can be negative (e.g., number or cat states)

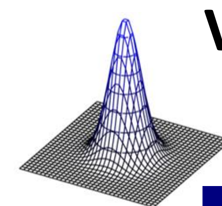
Vacuum fluctuations



$$\hat{H} = \hbar\omega_{\mathrm{r}} \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right)$$

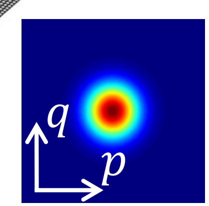
$$\langle \hat{a}^\dagger \hat{a} \rangle = 0$$

$$\Delta P = \Delta Q = \langle \hat{a}^\dagger \hat{a} + \frac{1}{2} \rangle = \frac{1}{2}$$



Vacuum

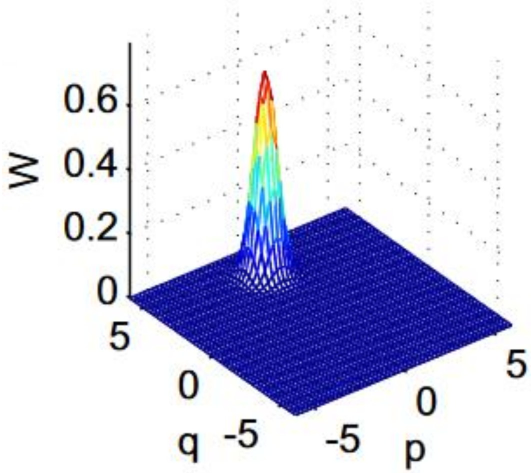
$|\mathbf{0}\rangle$



Gaussian CV states

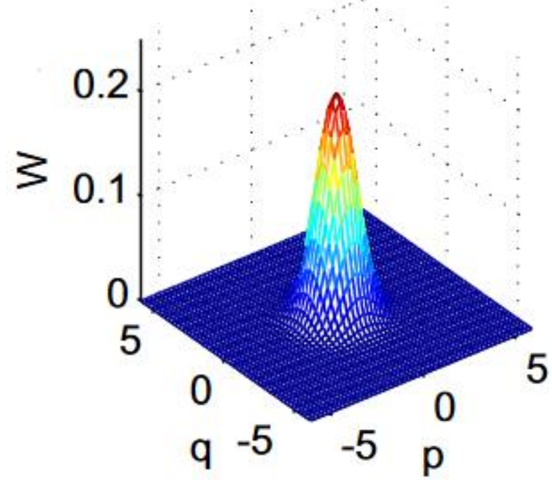
Gaussian states → Fully described by first two moments (covariance matrix; higher cumulants vanish)
→ Sections through maxima of $W(Q,P)$ Gauss-shaped
→ General: Displaced squeezd thermal state
→ Squeeze & displacement operations noncommuting

Coherent state



Displaced vacuum state
$(\Delta P)^2 = {}^1\!/_4$ and $(\Delta Q)^2 = {}^1\!/_4$

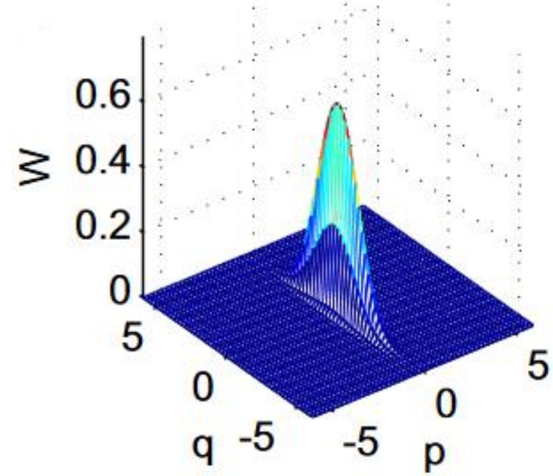
Thermal state ($\langle \hat{a}^\dagger \hat{a} \rangle = 1$)



Vanishing 1st moment
$(\Delta P)^2 > {}^1\!/_4$ and $(\Delta Q)^2 > {}^1\!/_4$

„classical“

Squeezed state



$(\Delta P)^2 \leq {}^1\!/_4$ XOR $(\Delta Q)^2 \leq {}^1\!/_4$

„quantum resource“

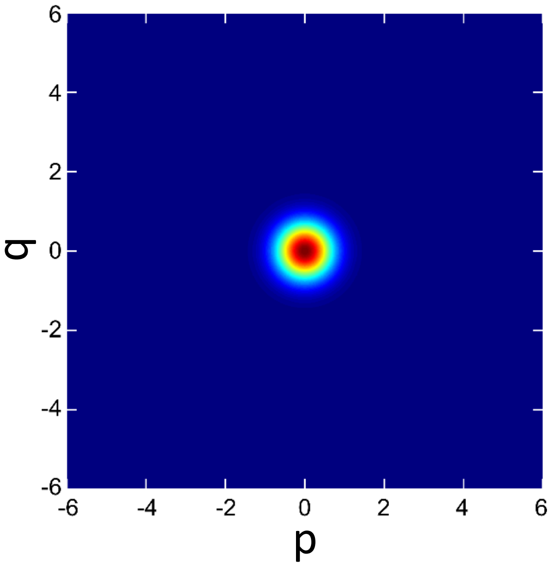
E. P. Menzel, PhD thesis (2013).

Squeezing (simulation)

Degenerate mode:

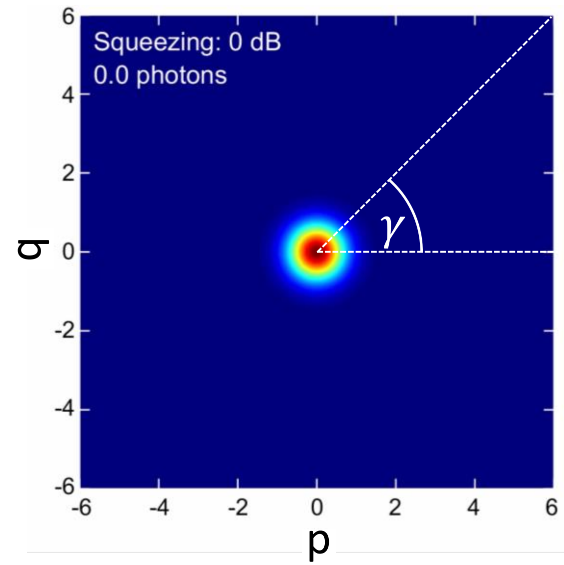
$f_{\mathrm{signal}} = f_{\mathrm{pump}}/2$

vacuum state



$$\hat{S}(\xi) = \exp\left(\frac{1}{2}\xi^*\hat{a}^2 + \frac{1}{2}\xi\left(\hat{a}^\dagger\right)^2\right)$$

squeezed vacuum state



squeezing level $\mathcal{S} = -10\log_{10}\left[(\Delta X_{\mathrm{sq}})^2/0.25\right]$

Number of photons $\langle \hat{a}^\dagger \hat{a} \rangle$ increases with increasing squeezing!

Propagating quantum microwaves

Theory

→ Propagating EM fields generally multi-mode

→ Each mode characterized frequency + wavevector

→ Quantized mode by mode

→ Here: mostly single-mode approximation

→ Microwaves: No polarization DOF in transmission lines

Experimental challenges

→ Protocols from optics don‘t consider low photon energy

→ Microwave components not tested for quantum signals

→ Required: Squeezing, tomography, beam splitting, displacement, entanglement, feedforward

→ Benchmark protocols: remote state preparation, teleportation

→ Microwave photodetection?

Envisioned applications of CV propagating quantum microwaves

Quantum communication

R. Di Candia *et al.*, EPJ Quantum Technology **2**, 25 (2015)

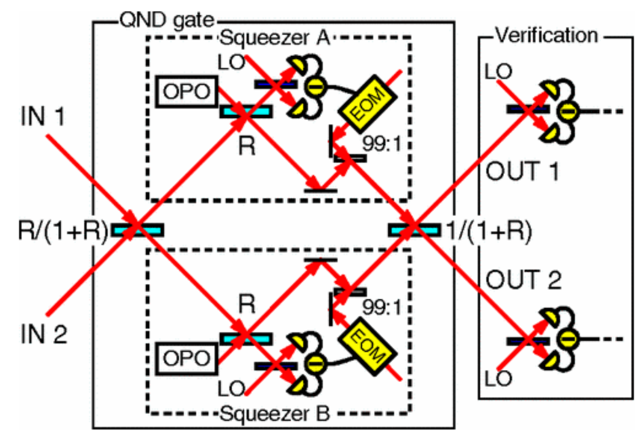
Quantum illumination

U. Las Heras *et al.*, arXiv:1611.10280 (2017)

Based on the same experimental toolbox!

Analog quantum computing

S. L. Braunstein and P. van Loock, Rev. Mod. Phys. 77, 513 (2005)

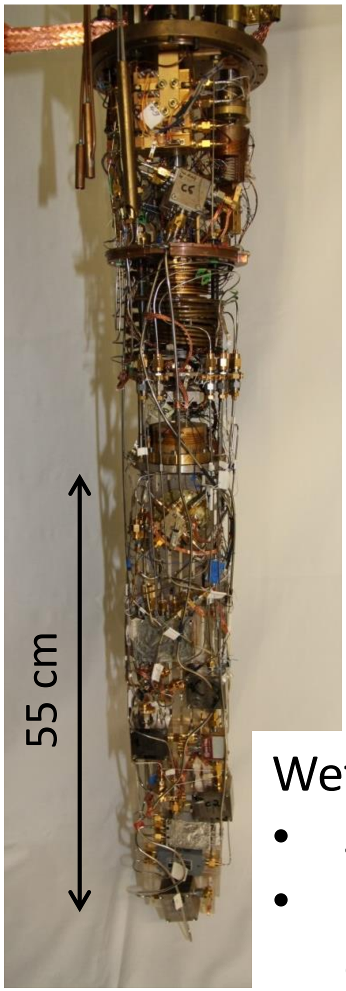


Digital quantum computing

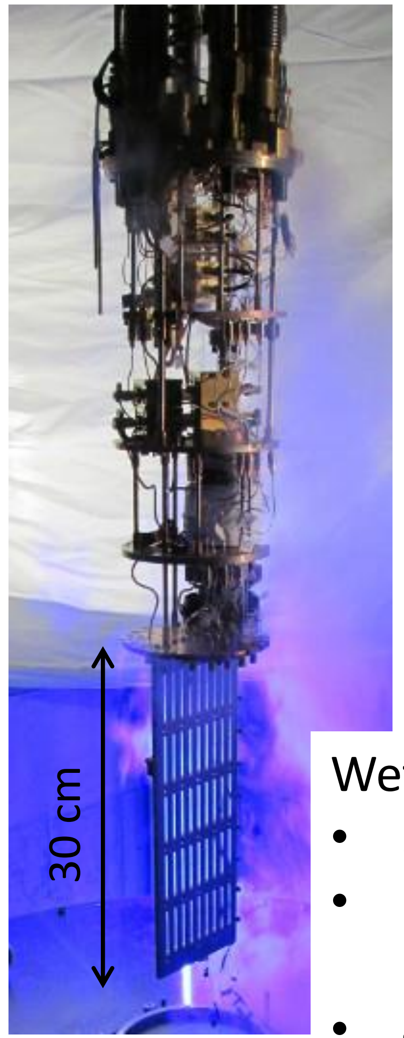
Phys. Rev. Lett. 101, 250501 (2008)

Cryogenics made @ WMI

Experimental results measured in three homemade dilution refrigerators



Wet cryostat
- Since 2007
- 11 coax cables
- 4 HEMTs

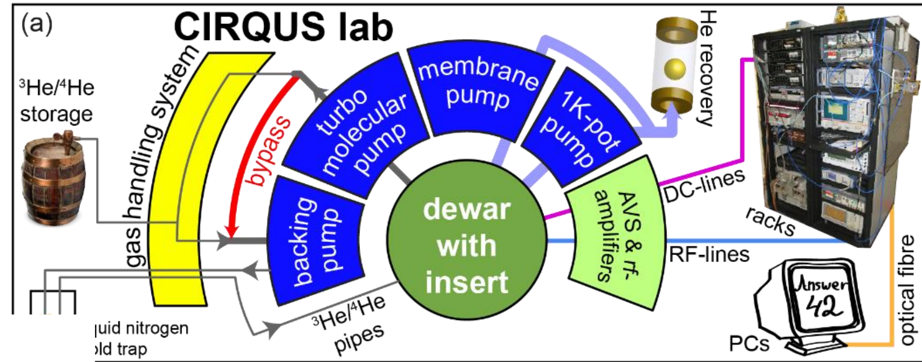


Wet cryostat
- Since 2013
- 24 coax cables
- 4-5 HEMTs

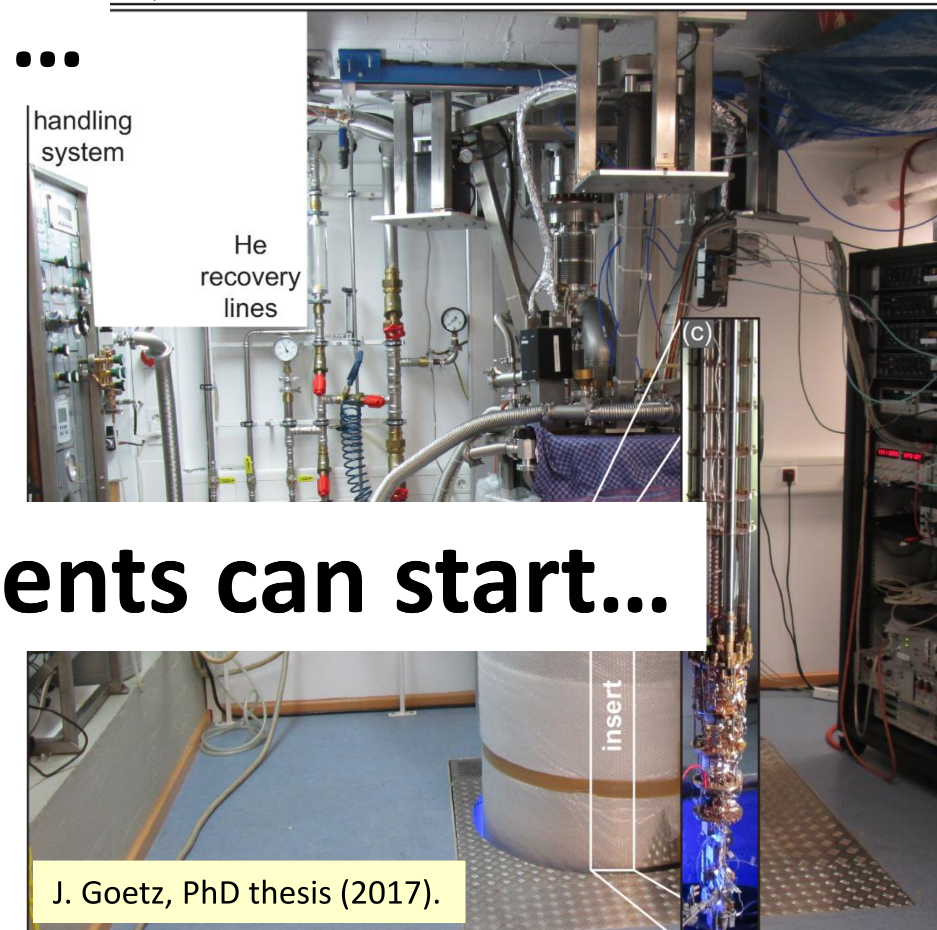
Dry cryostat
- since 2014
- 10+ coax cables
- 4-6 HEMTs

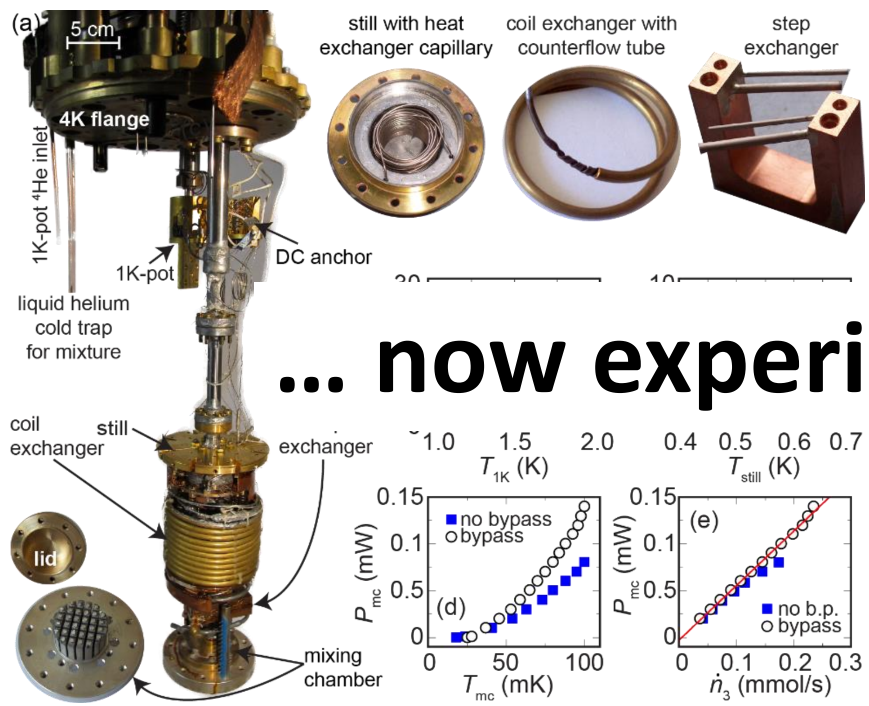
Life as a PhD student in low-temperature physics

Lab @ beginning of Jan‘s PhD



... 2.5 yrs ...

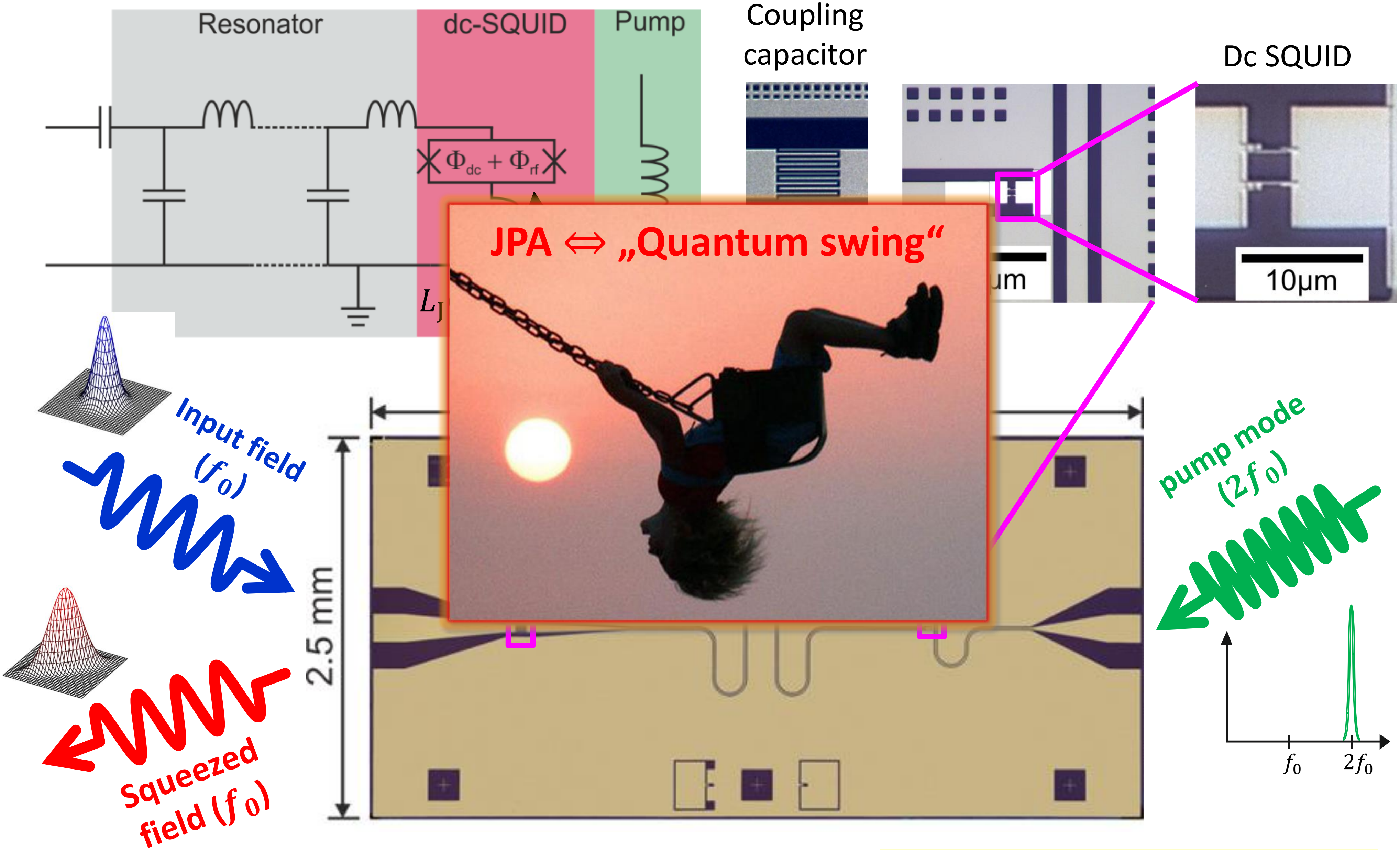




... now experiments can start...

J. Goetz, PhD thesis (2017).

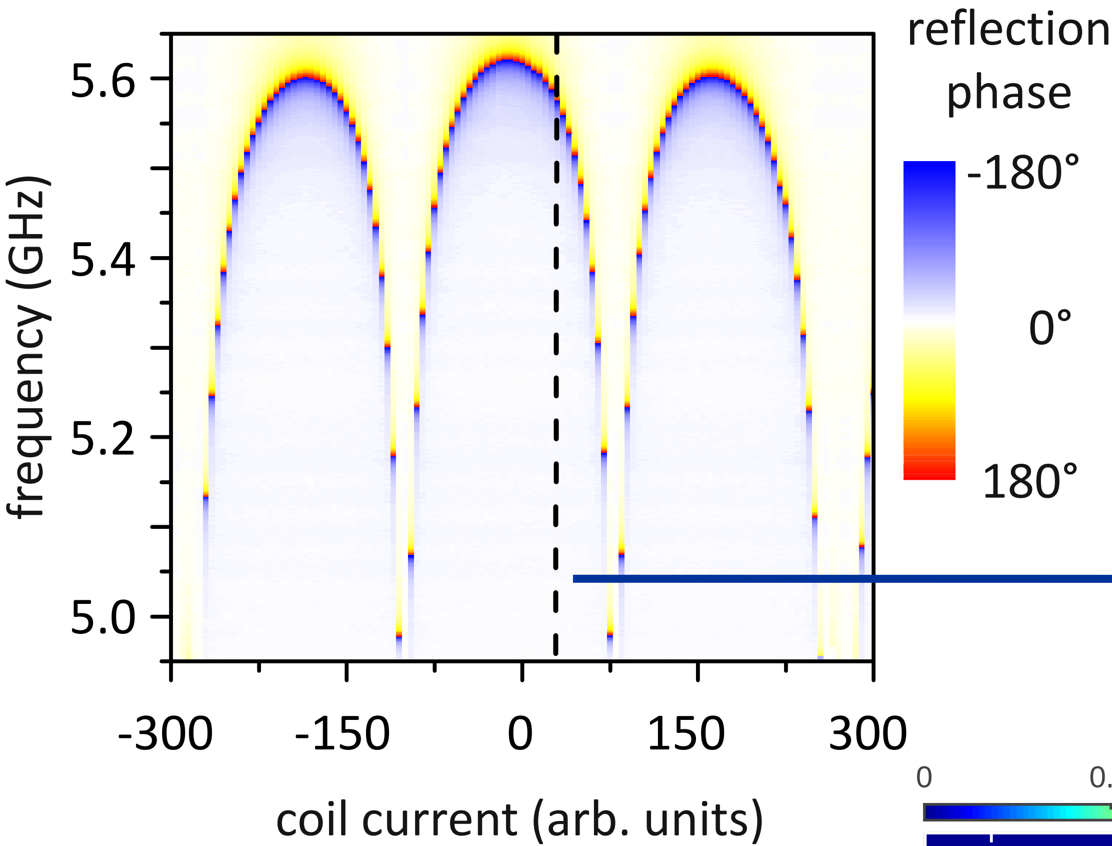
Flux-driven Jospehson parametric amplifier

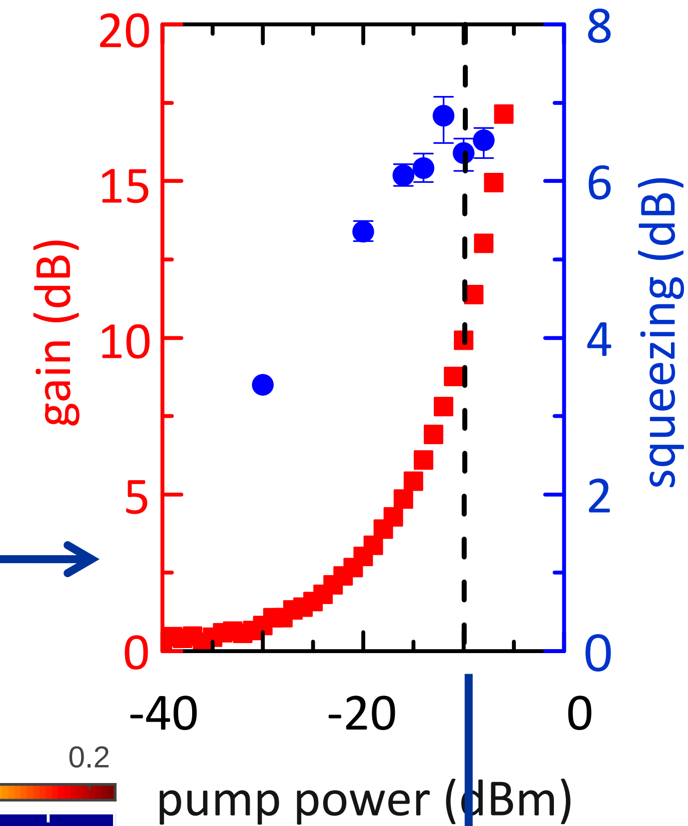


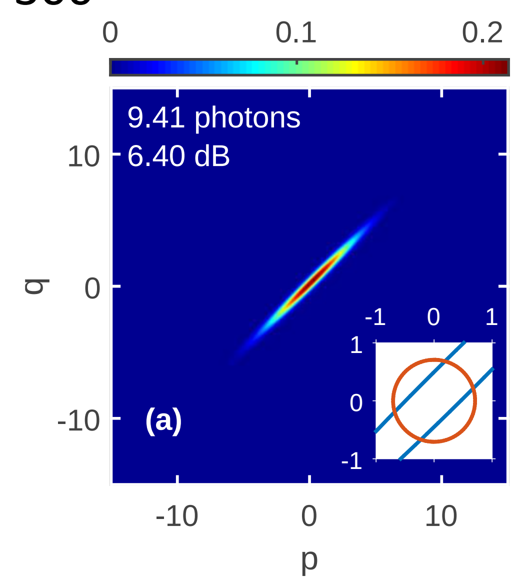
T. Yamamoto et al., APL 93, 042510 (2008).

L. Zhong *et al.*, New. J. Phys. **15**, 125013 (2013).

Squeezing & nondegenerate gain







$T_{noise} \approx 300$ mK

$$\mathcal{S} = -10 \log_{10}\left[(\Delta X_{sq})^2/0.25\right]$$

S. Pogorzalek *et al.*, Phys. Rev. Applied 8, 024012 (2017).
K. G. Fedorov *et al.*, PRL **117**, 020502 (2016).

Quantum state tomography

→ Greek: tomos = slice, section; grapho = to write

→ Write a complete image by sectioning

→ QST: Complete description of a quantum state

→ Denisty matrix, Wigner function, characteristic function…

→ States of light: typically $W(Q,P)$

Quantum state tomography of propagating microwaves

Quantum microwaves

Photodetection
(difficult, only recent progress)

Linear amplifiers
Low, but finite added noise
(0.5-20 photons)

Signal recovery methods
(successful in classical radio atronomy or radar)

Phase insensitive parametric amplifiers
(demanding in practice)

Dual-path state tomography (WMI)

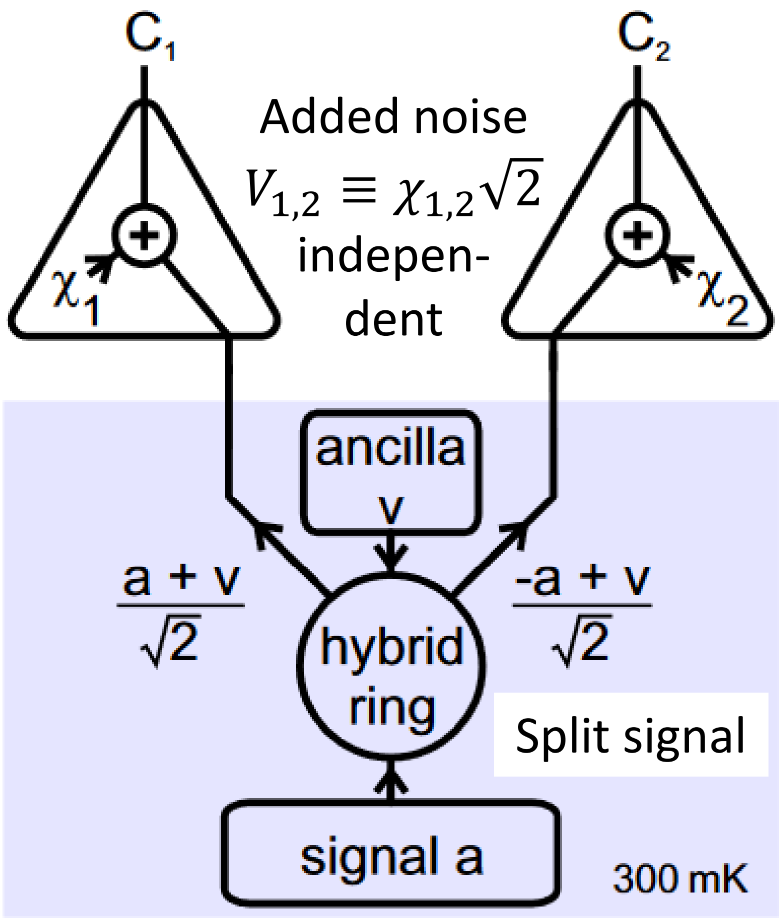
E. P. Menzel *et al*., Phys. Rev. Lett. **105**, 100401 (2010).
R. Di Candia *et al.*, New J. Phys. **16**, 015001 (2014).

Reference-state tomography (ETHZ)

M. P. da Silva *et al*., Phys. Rev. A **82**, 043804 (2010).

Dual-path state tomography (classical)

Cross correlations $\langle C_1^n C_2^m \rangle$
→ Signal survives, noise cancels



E. P. Menzel *et al.*, Phys. Rev. Lett. **105**, 100401 (2010).

→ Evident for 1st & 2nd signal moment
$\langle C_{1,2}^2 \rangle$ contain $\langle V_{1,2}^2 \rangle > 0$
$\langle C_1 C_2 \rangle$ contains only $\langle V_1 V_2 \rangle = 0$

→ Ancilla v vaccum or thermal state

→ Assume equal power gain G in both amplification chains (simpler formulas)

→ Higher moments by induction

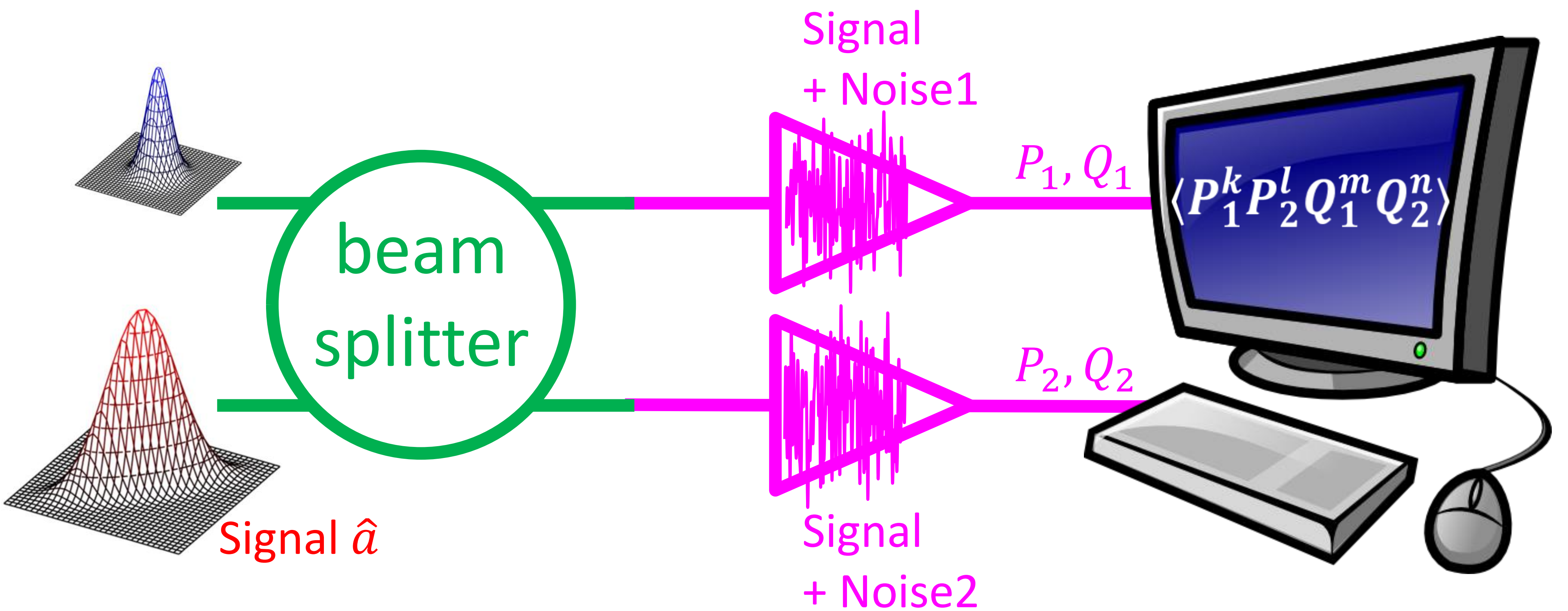
$$\langle a^n \rangle = -\langle C_1^{n-1} C_2 \rangle / G^{\frac{n}{2}}$$
$$-\sum_{k=1}^{n-1}\sum_{j=0}^{k}\binom{n-1}{k}\binom{k}{j}\langle a^{n-k}\rangle\langle v^j\rangle\langle V_1^{n-k}\rangle$$
$$+\sum_{k=1}^{n-1}\sum_{j=0}^{k}\binom{n-1}{k}\binom{k}{j}\langle a^{n-k-1}\rangle\langle v^{j+1}\rangle\langle V_1^{n-k}\rangle$$

(Right side → only terms of order $< n-1$)

→ Similar formulas for noise moments $\langle V_{1,2}^n \rangle$

Dual-path state tomography (quantum)

Straightforward extension from classical treatment → $C \to \hat{Q} + i\hat{P}$



Correlations $\langle P_1^k P_2^l Q_1^m Q_2^n \rangle$ ➡ All signal moments up to $\langle \hat{a}^j \rangle, \langle (\hat{a}^\dagger)^j \rangle$ for $k + l + m + n \leq j$

Gaussian states → 2nd-order moments for reconstruction
4th-order moments for verification (cumulants)

R. Di Candia *et al.*, New J. Phys. **16**, 015001 (2014).

Reference state tomography

Dual-path state tomography

Self-calibrated
but significant experimental overhead

Single path
Possible if noise moments pre characterized against reference state

Fast signal pulsing
Signal obtained from deconvolution („algebraic inversion“)

~~Measure noise once & deconvolute~~
Amplifiers drift slowly

Reference-state tomography

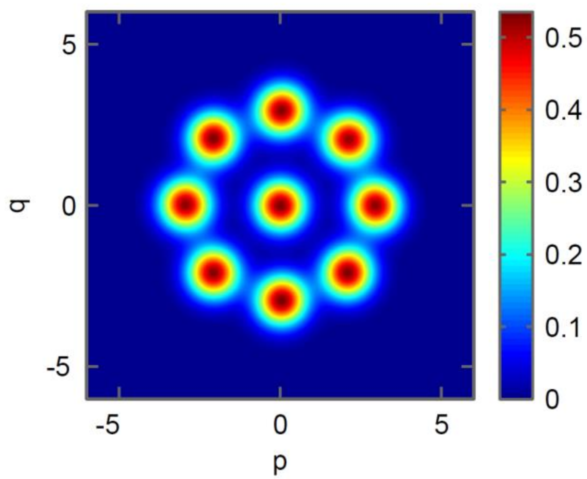
M. P. da Silva *et al.*, Phys. Rev. A **82**, 043804 (2010).

Less experimental resources required
Comparatively slow

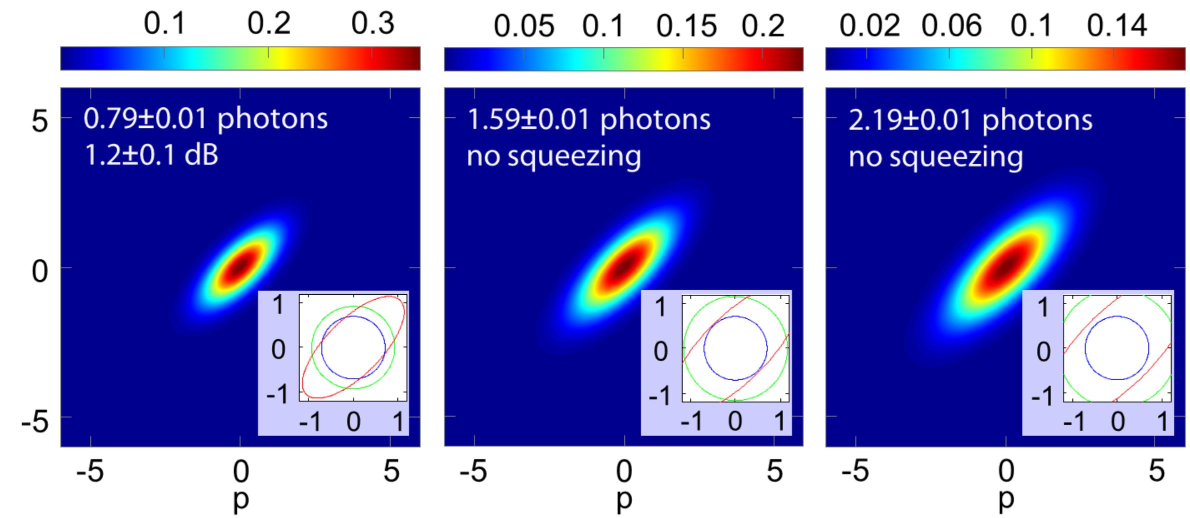
R. Di Candia *et al.*, New J. Phys. **16**, 015001 (2014).

Dual-path state reconstructions

Vacuum & coherent states

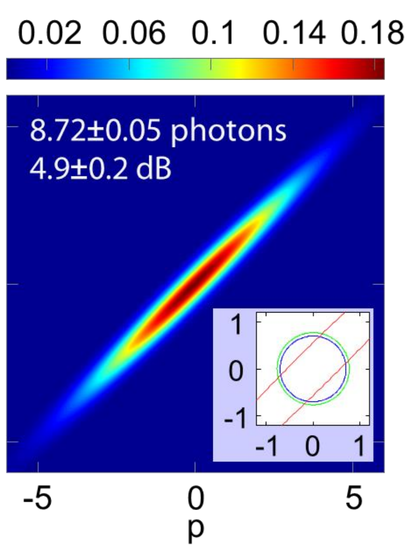


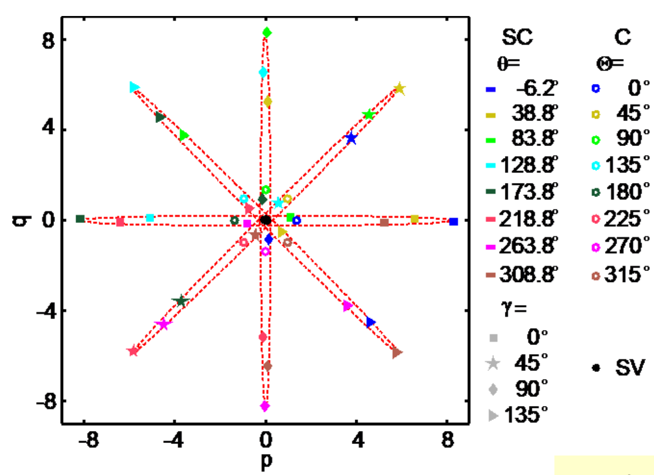
Squeezed thermal states



Temperature → 210 mK 412 mK 573 mK

Squeezed vacuum

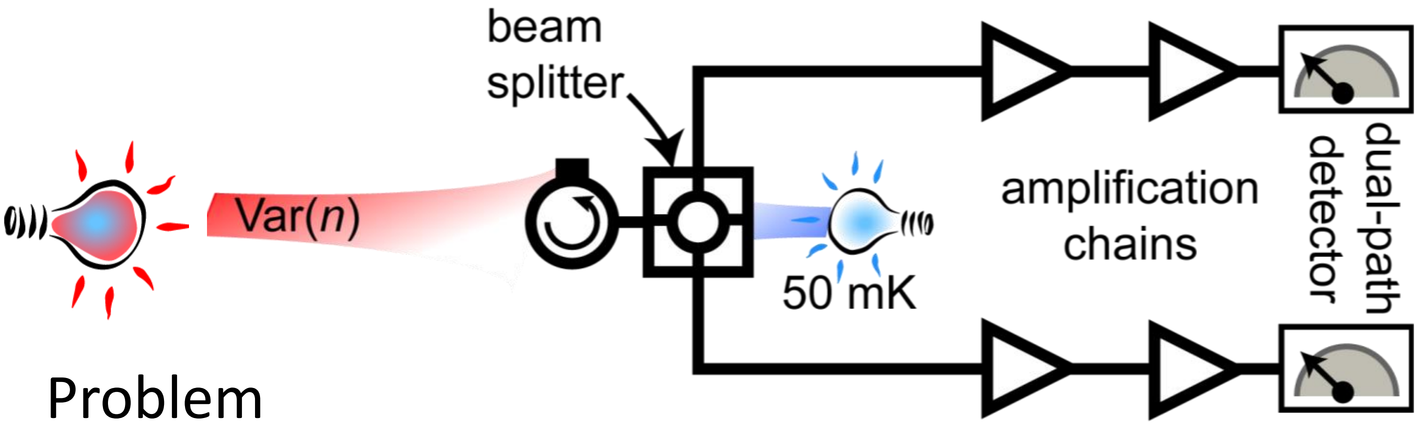


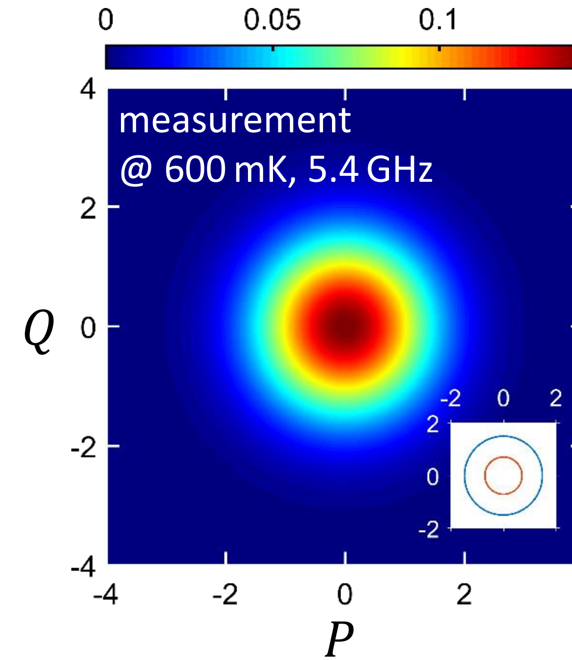


Squeezed coherent states

L. Zhong *et al.*, New. J. Phys. **15**, 125013 (2013).
E. P. Menzel *et al.*, Phys. Rev. Lett. **105**, 250502 (2010).

Technical „detail“: Calibration





Problem

→ We measure amplified power P_{exp}

→ We plot photon numbers

→ Proportionality „photon number conversion factor (PNCF)“

Solution: „Planck spectroscopy“ M. Mariantoni *et al.*, Phys. Rev. Lett. **105**, 133601 (2010).

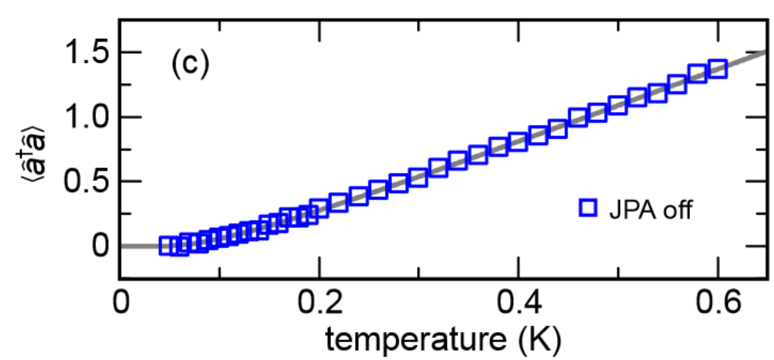
→ PNCF essentially determined by gains & losses

→ Exact calibration requires well-known cold source

→ WMI uses a black body emitter (thermally weakly coupled heatable 50Ω load)

→ $P_{\mathrm{exp}} = \mathrm{G} \times \mathrm{BW} \times \left[\hbar\omega\left(\langle \hat{a}^\dagger \hat{a}\rangle + \frac{1}{2}\right) + k_{\mathrm{B}} T_{\mathrm{noise}}\right]$

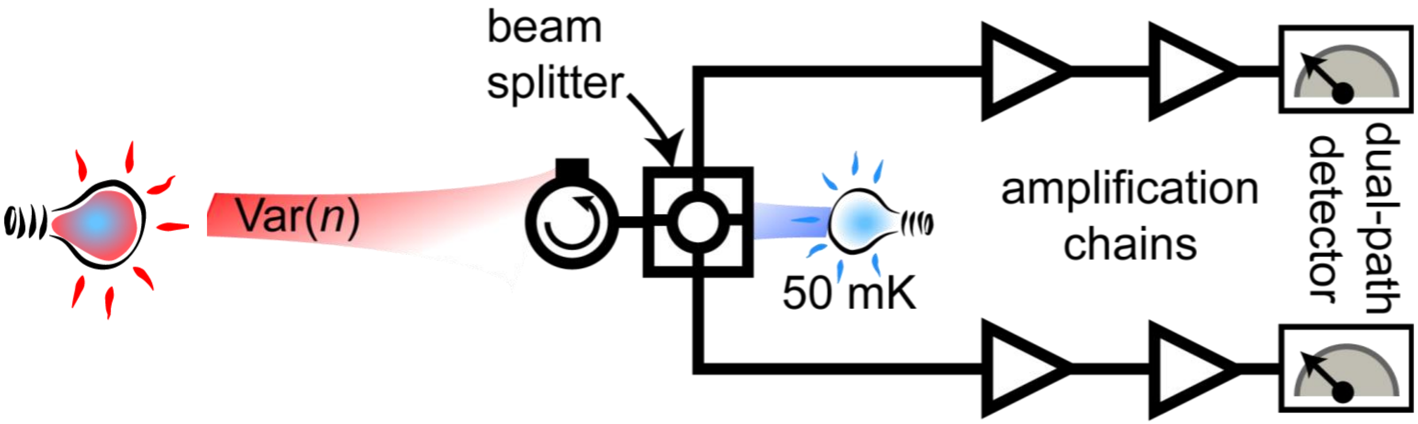
→ Bose-Planck distribution $\langle \hat{a}^\dagger \hat{a}\rangle = \frac{1}{\exp\left(\frac{\hbar\omega}{k_{\mathrm{B}}T}\right) - 1}$

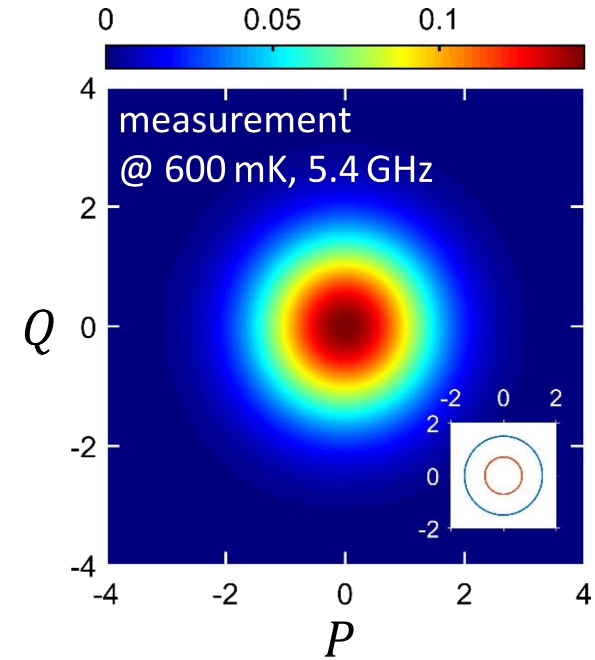


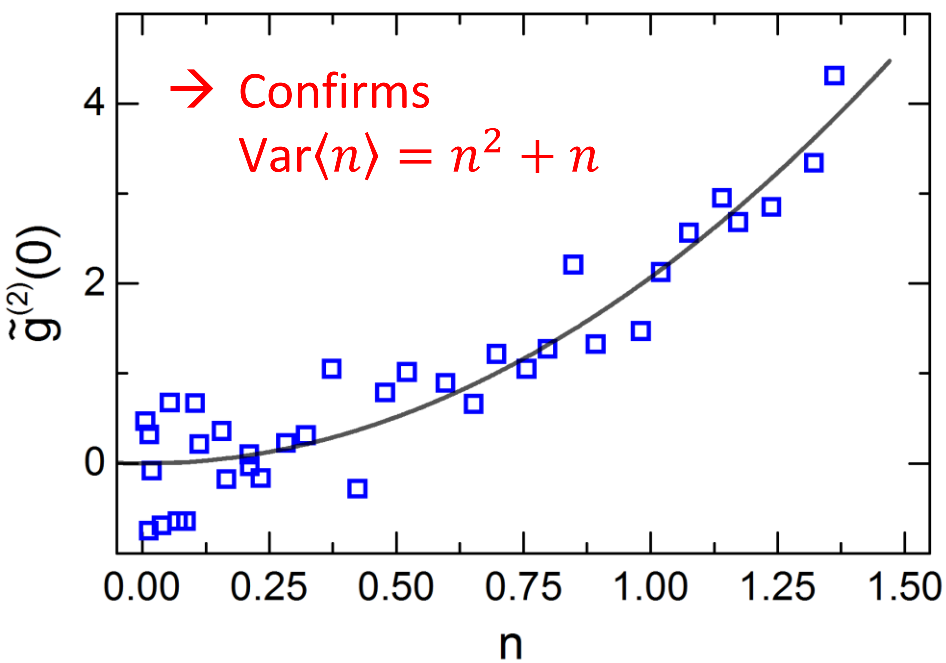
$T_{\mathrm{noise}} \approx T_{HEMT} \simeq 3\ \mathrm{K}$

J. Goetz *et al.*, Phys. Rev. Lett. **118**, 103602 (2017).

Side track: Photon statistics of thermal propagating microwaves







4th order moments

→ Photon-photon correlations?

$$g^{(2)}(0) = \frac{\mathrm{Var}\langle n\rangle - \langle n\rangle}{\langle n\rangle^2} + 1$$

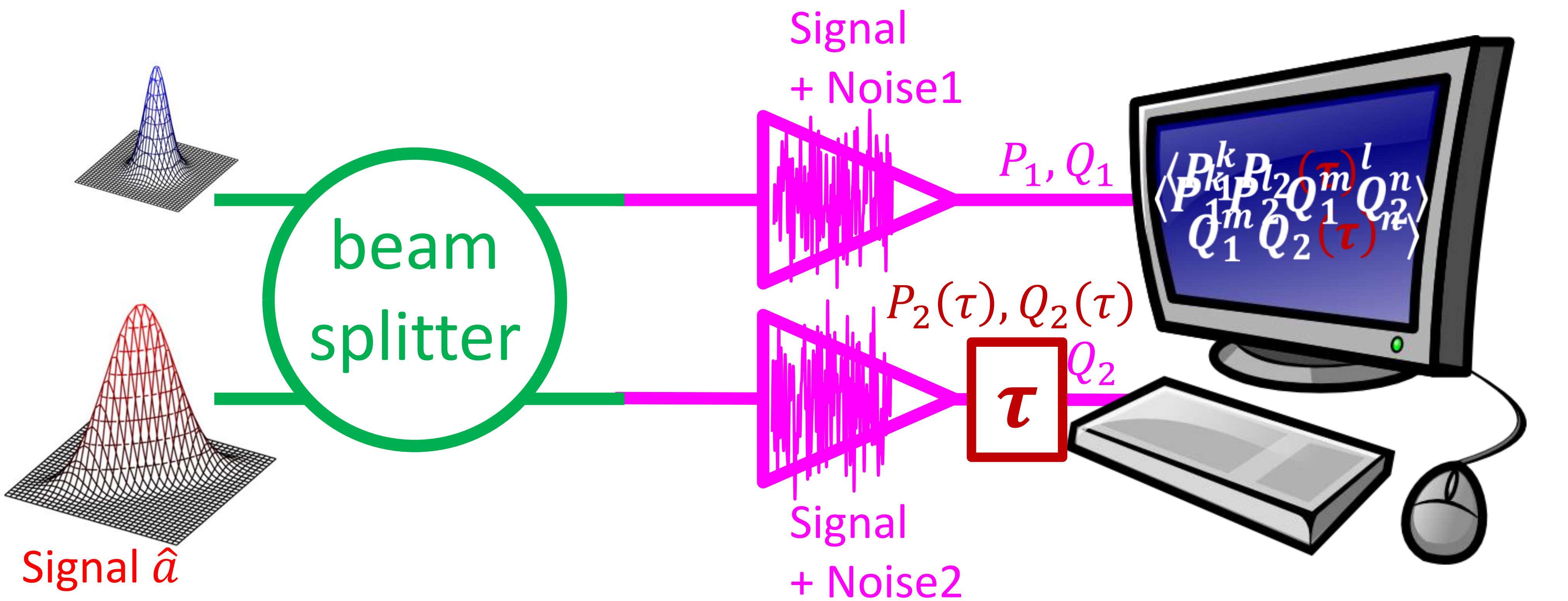
For thermal states:

$$\tilde{g}^{(2)} = \langle n\rangle^2 g^{(2)} = 2\langle n\rangle^2$$

J. Goetz *et al.*, Phys. Rev. Lett. **118**, 103602 (2017).

Finite-time intensity correlations of squeezed microwaves

Photon statistics à la dual-path → Finite-time photon-photon correlations

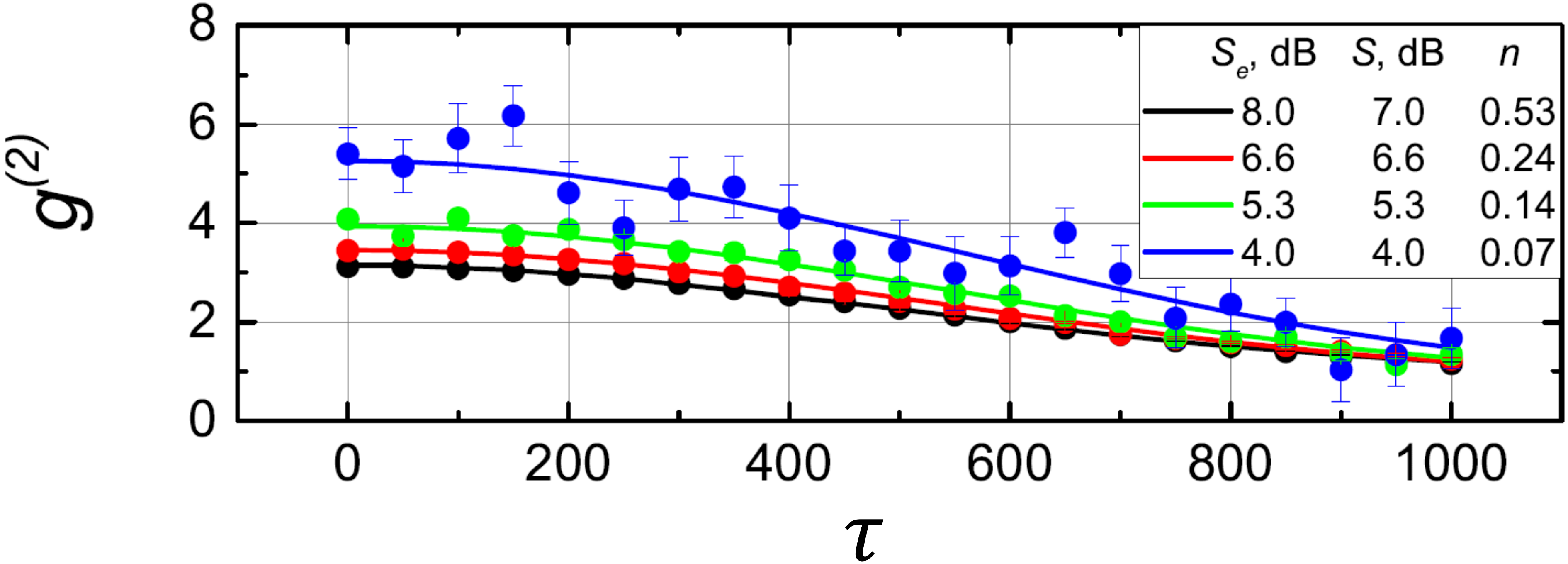


From moments up to 4th-order

$$g^{(2)}(\tau) = \frac{\langle \hat{a}^\dagger(0)\hat{a}^\dagger(\tau)\hat{a}(0)\hat{a}(\tau)\rangle}{\langle \hat{a}^\dagger \hat{a}\rangle^2}$$

E. P. Menzel *et al.*, Phys. Rev. Lett. **105**, 100401 (2010).
L. Zhong *et al.*, New. J. Phys. **15**, 125013 (2013).
K. G. Fedorov, S. Pogorzalek *et al.*, Sci. Rep. **8**, 6416 (2018).

Finite-time intensity correlations of squeezed microwaves



→ Bunching as expected ($g^{(2)}(0) \rightarrow 3$ for $S \rightarrow \infty$)
→ Coherent-state limit $g^{(2)}(\tau) \rightarrow 1$ reproduced for $\tau \rightarrow \infty$
→ Shape mostly defined by measurement bandwidth (well understood)

K. G. Fedorov, S. Pogorzalek *et al.*, Sci. Rep. **8**, 6416 (2018).

Intensity correlations of single-mode squeezed states

$$g^{(2)}(\tau) = 1 + \mathrm{sinc}^2(\omega\tau)\frac{1 + 2\sigma_x(\sigma_x - 1) + 2\sigma_p(\sigma_p - 1)}{(1 - \sigma_x - \sigma_p)^2}$$

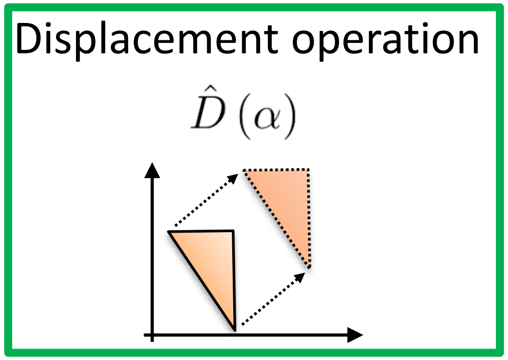
↑ measurement bandwidth

variances:

$$\sigma_p = \frac{1}{(2\chi + \kappa + \gamma)^2}[(2\chi - \kappa + \gamma)^2(n_{b_{in}} + 1/2) + 4\kappa\gamma(n_{c_{in}} + 1/2)]$$

$$\sigma_x = \frac{1}{(2\chi - \kappa - \gamma)^2}[(2\chi + \kappa - \gamma)^2(n_{b_{in}} + 1/2) + 4\kappa\gamma(n_{c_{in}} + 1/2)]$$

Displacement operation (theory)



Displacement operator: $\widehat{D}(\alpha) = \exp\left(\alpha\hat{a}^{\dagger} - \alpha^{*}\hat{a}\right)$

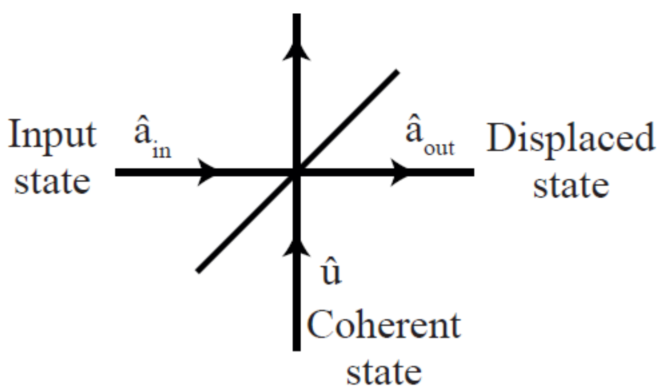
α = Displacement vector in phase space

$$\hat{a}_{\text{out}} = \sqrt{\tau}\hat{a}_{\text{in}} + \sqrt{\nu}\hat{u}$$

$$\tau \to 1 \qquad \hat{a}_{\text{out}} = \hat{a}_{\text{in}} + \sqrt{\nu}\hat{u}$$

$$\widehat{D}^{\dagger}(\alpha)\hat{a}_{\text{in}}\widehat{D}(\alpha) = \hat{a}_{\text{in}} + \alpha \text{ with } \alpha = \sqrt{\nu}u$$

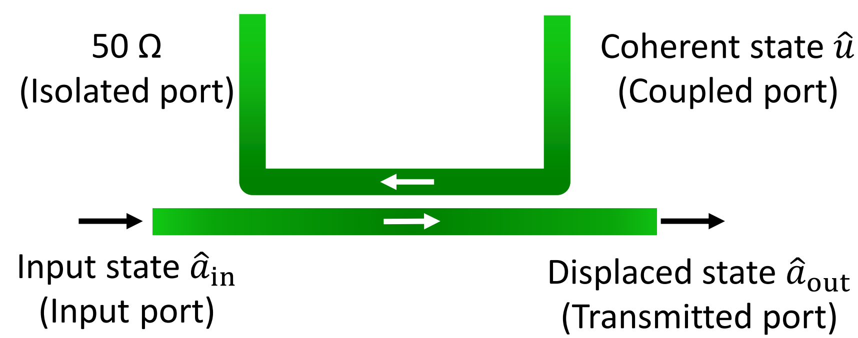
Optics → 99% transmissive beam splitter



τ - power linear transmissivity

ν - power linear reflectivity

Microwaves → Directional coupler

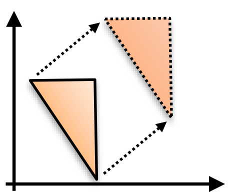


→ Displacement is CV quantum gate & required in feedforward schemes

Displacement operation (Simulation)

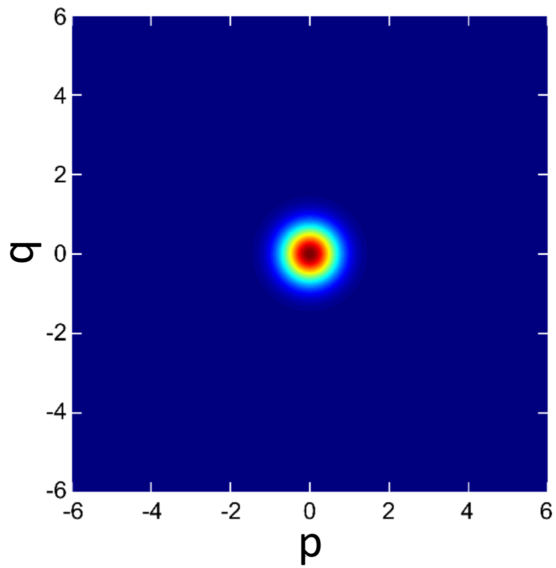
Displacement operation

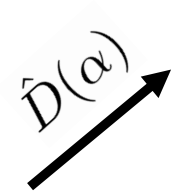
$\hat{D}(\alpha)$



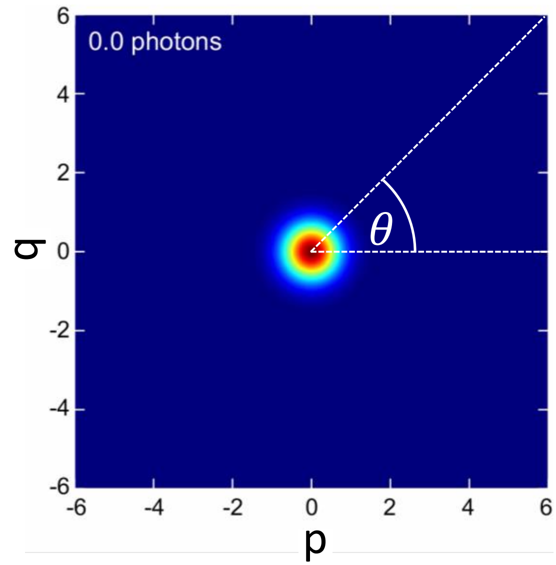
Displacement operator: $\widehat{D}(\alpha) = \exp\left(\alpha \hat{a}^{\dagger} - \alpha^{*}\hat{a}\right)$

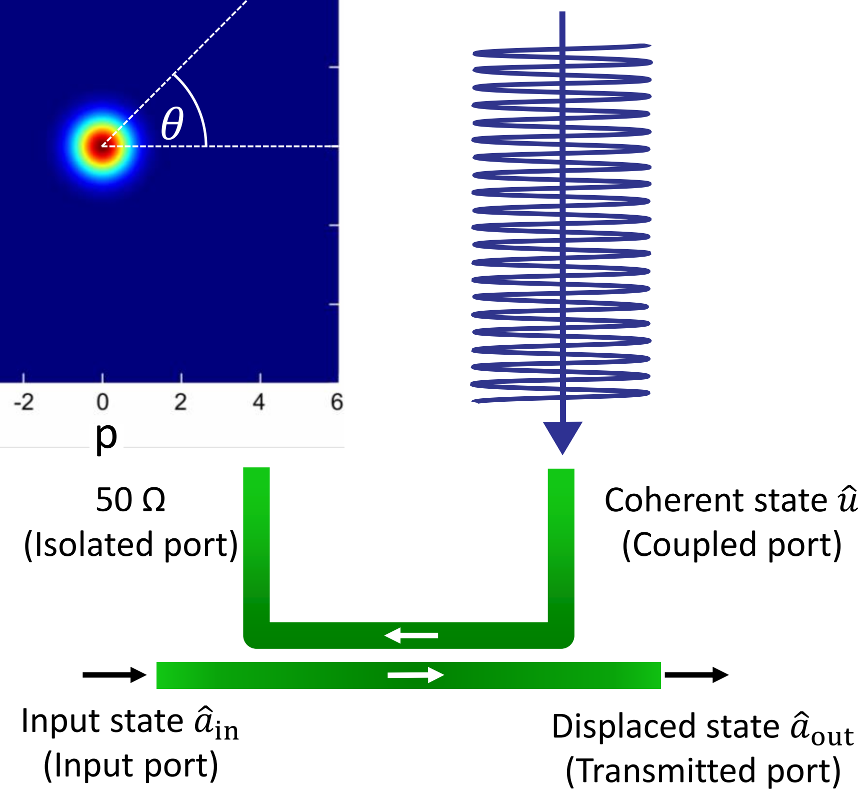
Vacuum state





Displaced vacuum state

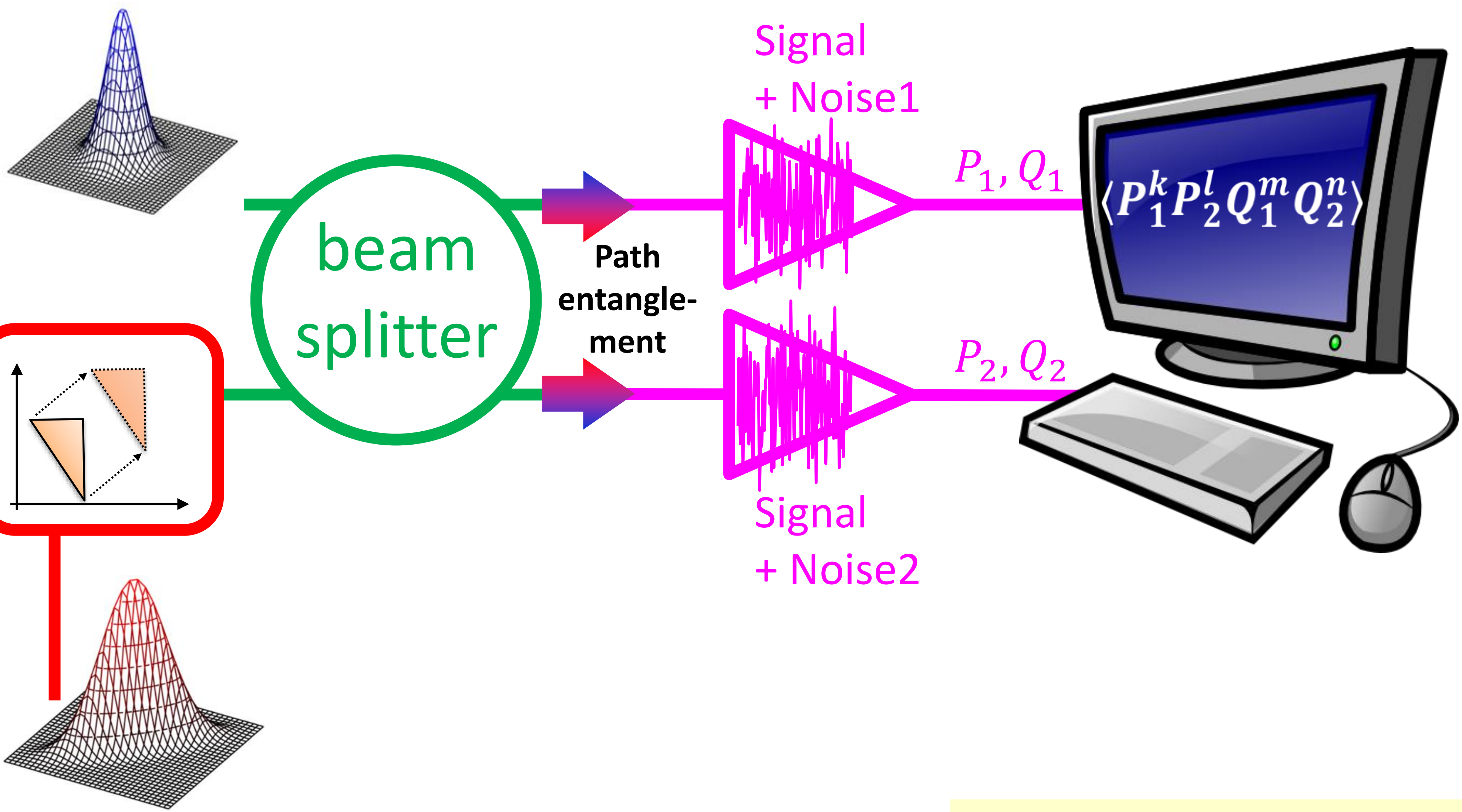




How about actual quantum states?

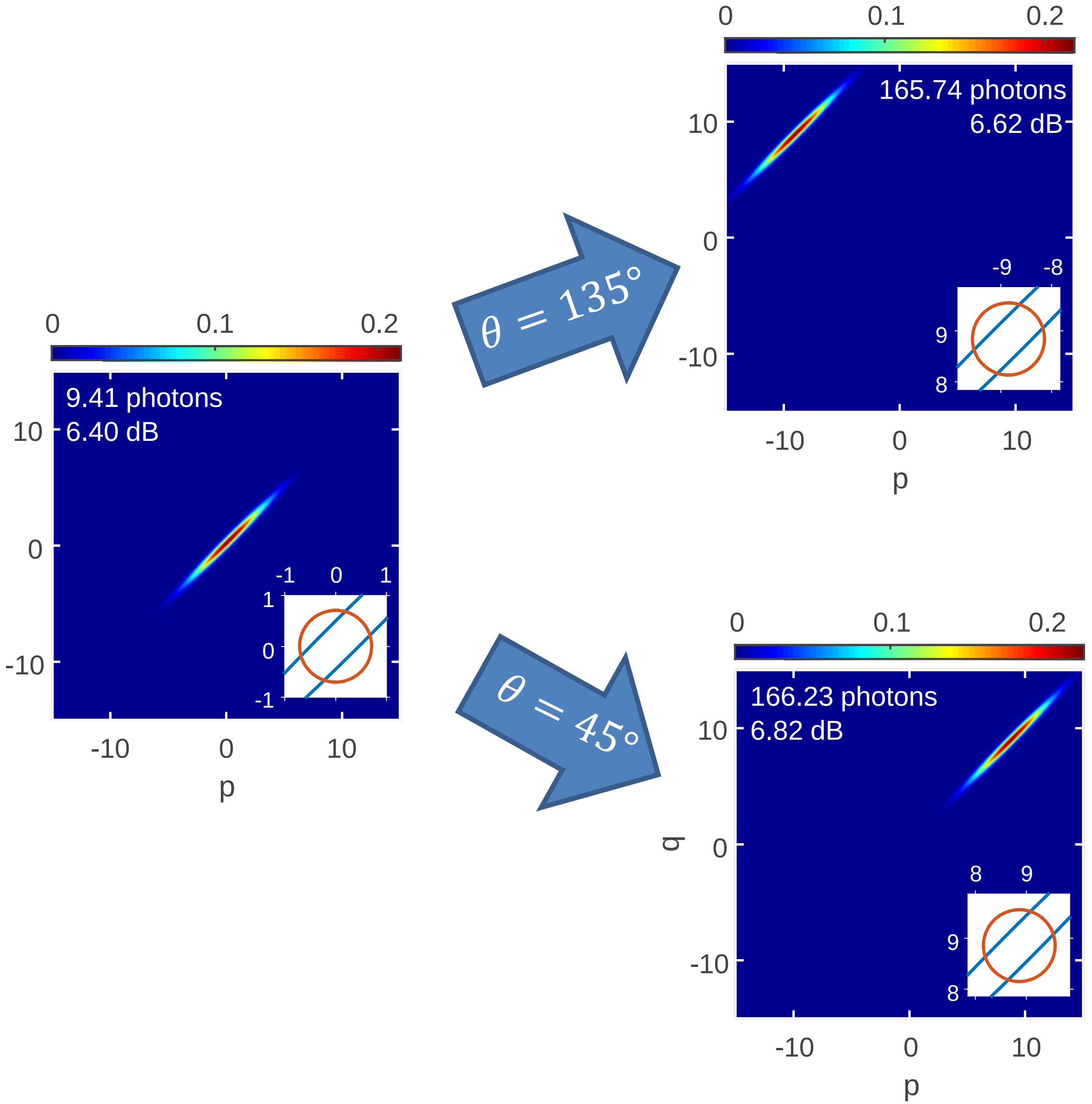
Measuring displacement of propagating microwaves

Directional coupler acts as displacer

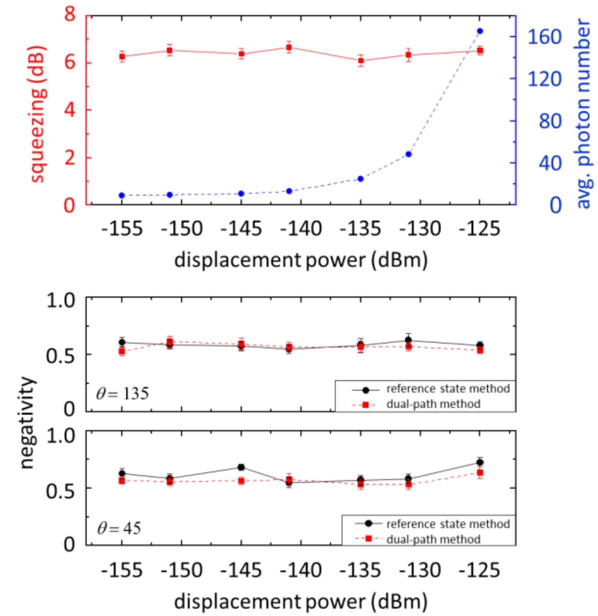


K. G. Fedorov *et al.*, PRL **117**, 020502 (2016).

Displacement of squeezed microwaves



→ High degree of control over angle and magnitude
→ Hundreds of displacement photons referred to 400 kHz bandwidth
→ Squeezing and negativity nearly unchanged



K. G. Fedorov *et al.*, PRL **117**, 020502 (2017).

Entanglement

Bipartite entaglement

→ Nonlcassical correlations between two subsystems

→ Defined via non-separability

  → Positive partial transpose (PPT) criterion

  → Separable state → PPT has positive eigenvalues

  → Negative eigenvalues → Non-separable → Entanglement

→ Witness functions indicate presence (not absence!)

→ In some situations quantitative measures exist

Bipartite entanglement in Gaussian states

→ Quantum correlations between quadratures of subsystems („two-mode squeezing“)

→ Measures: Negativity, log negativity, entanglement of formation…

→ Negativity $\mathcal{N} = \max\{\tilde{\mathcal{N}}; 0\} > 0$ → Entanglement

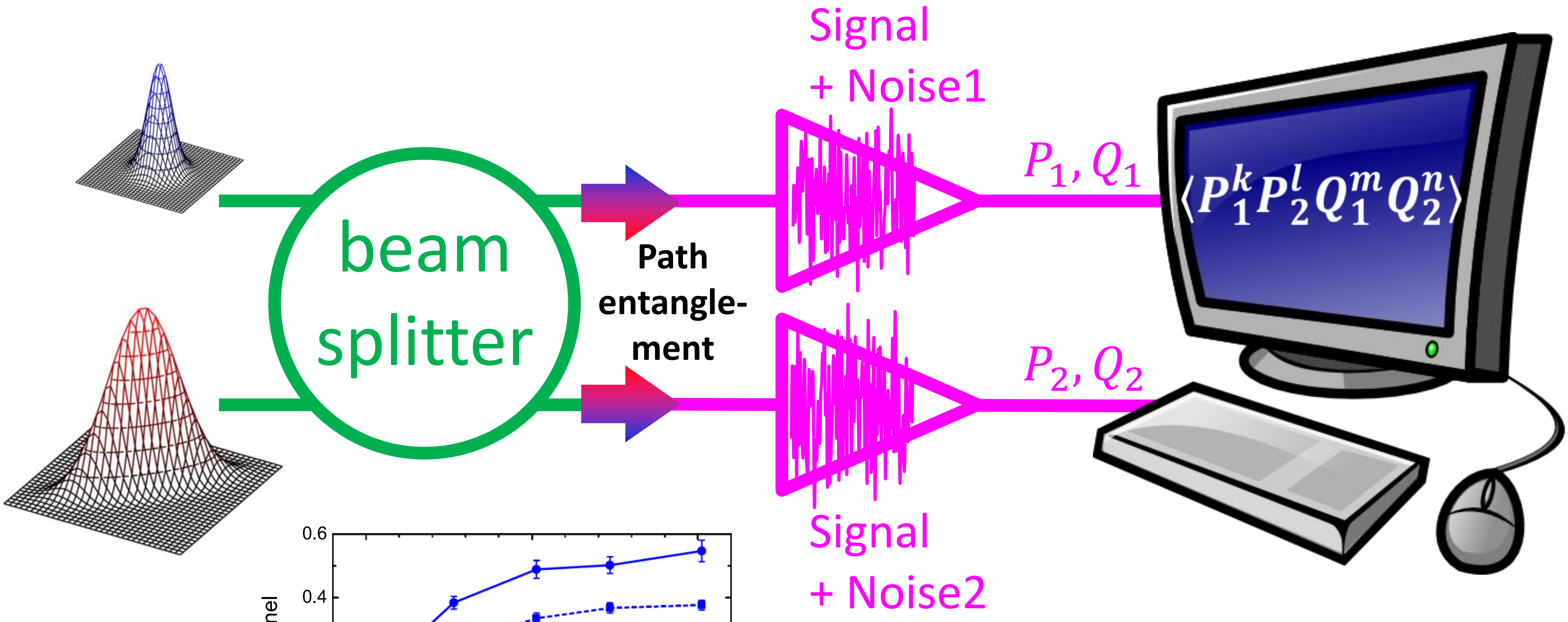
$$\tilde{\mathcal{N}} = \frac{1-\nu}{2\nu} \qquad \nu \equiv \sqrt{\left(\Delta(\boldsymbol{\sigma}) - \sqrt{\Delta^2(\boldsymbol{\sigma}) - 4\det\boldsymbol{\sigma}}\right)/2} \qquad \Delta(\boldsymbol{\sigma}) \equiv \det\boldsymbol{\alpha} + \det\boldsymbol{\beta} - 2\det\boldsymbol{\gamma}$$

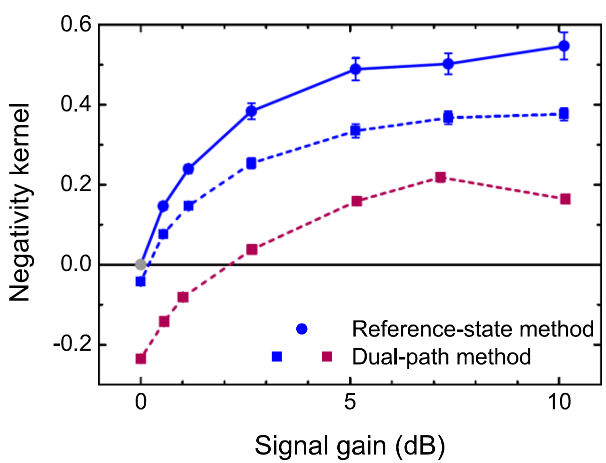
→ Computed from covariance matrix $\boldsymbol{\sigma}$ based on 1st & 2nd order moments

→ Experimentally, negativity kernel $\tilde{\mathcal{N}}$ plotted (difficult to measure 0)

CV path entanglement

Analyze **two-mode output** of beam splitter with reference state tomography





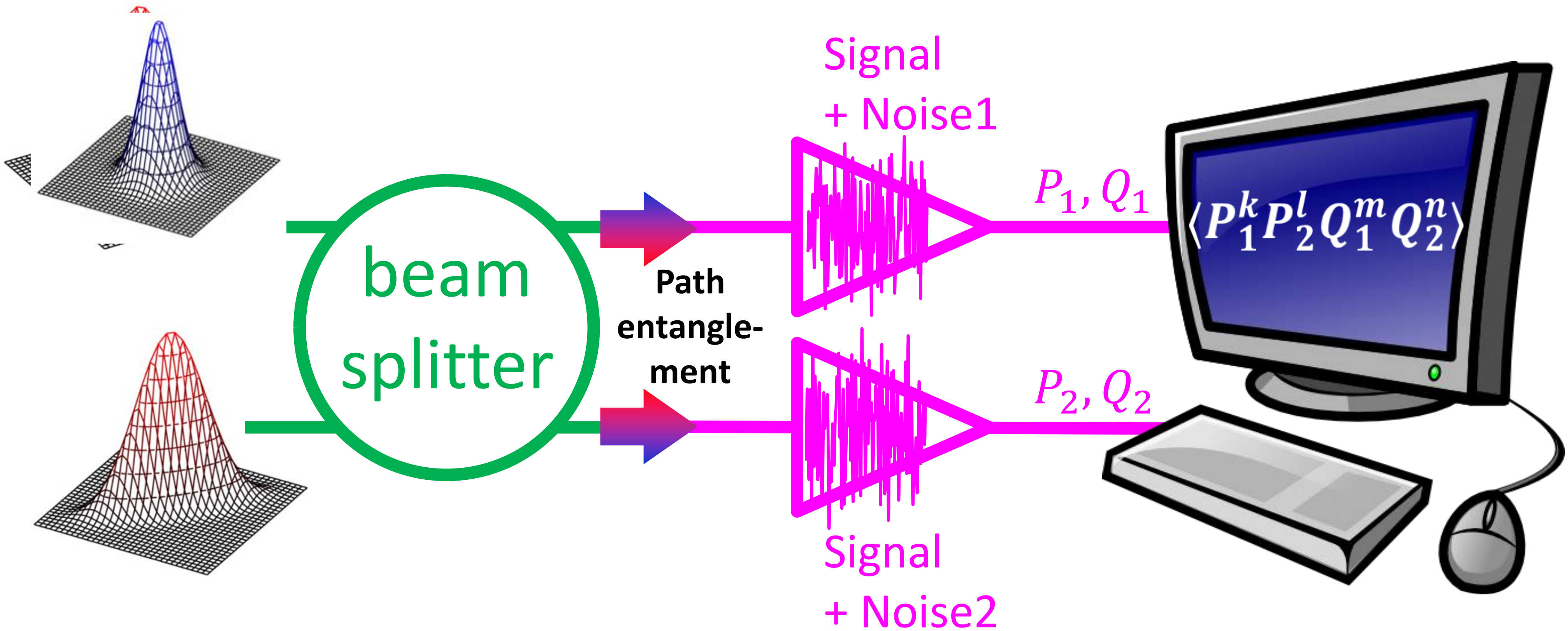
→ Maximum negativity 0.55 ↔ **3.2 dB TMS**

→ Still residual single-mode squeezing

E. P. Menzel *et al.*, Phys. Rev. Lett. **109**, 250502 (2012).

Tomography of two-mode squeezing

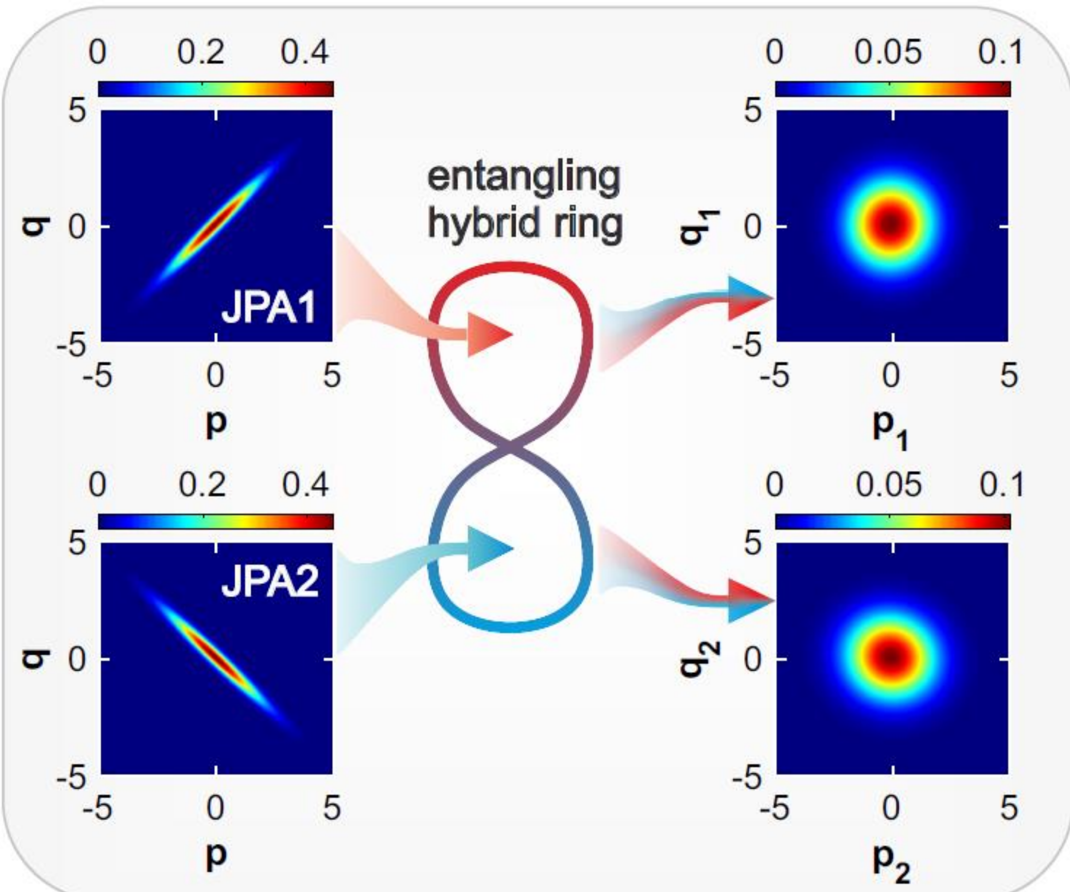
Vacuum input leads to residual single-mode squeezing → No EPR state



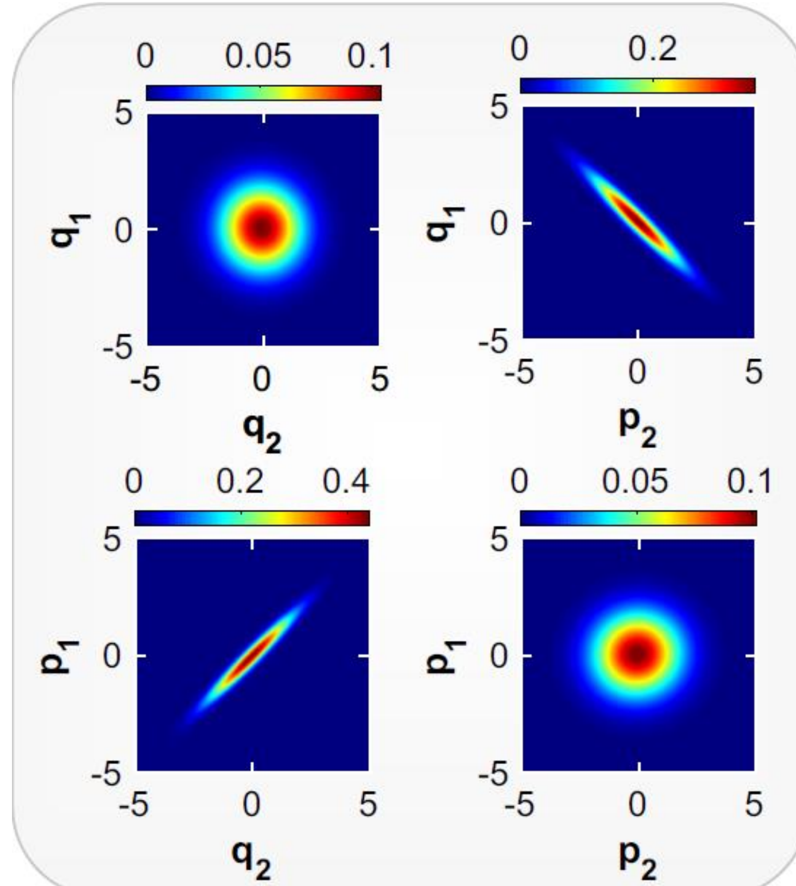
→ Two-mode squeezing should depend on the relative angle
→ **Pure/balanced two-mode squeezing** expected for 90°
→ Correlation measurement allows for tomography of two-mode state

K. G. Fedorov, S. Pogorzalek *et al.*, Sci. Rep. **8**, 6416 (2018).

Tomography of two-mode squeezing



Balanced two-mode squeezing



No information in single paths

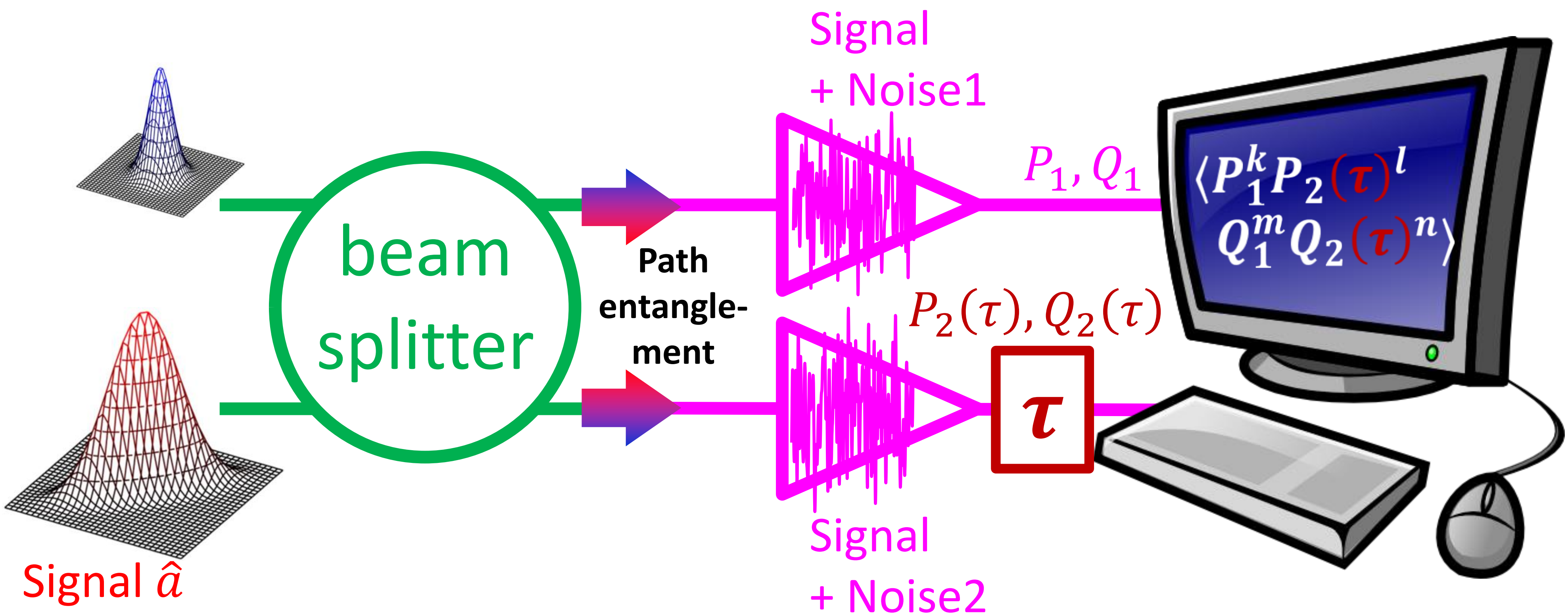
All information in entanglement

→ Ressource for CV quantum communication

K. G. Fedorov, S. Pogorzalek *et al.*, Sci. Rep. **8**, 6416 (2018).

Finite-time entanglement correlations of squeezed microwaves

Photon statistics à la dual-path → Finite-time photon-photon correlations

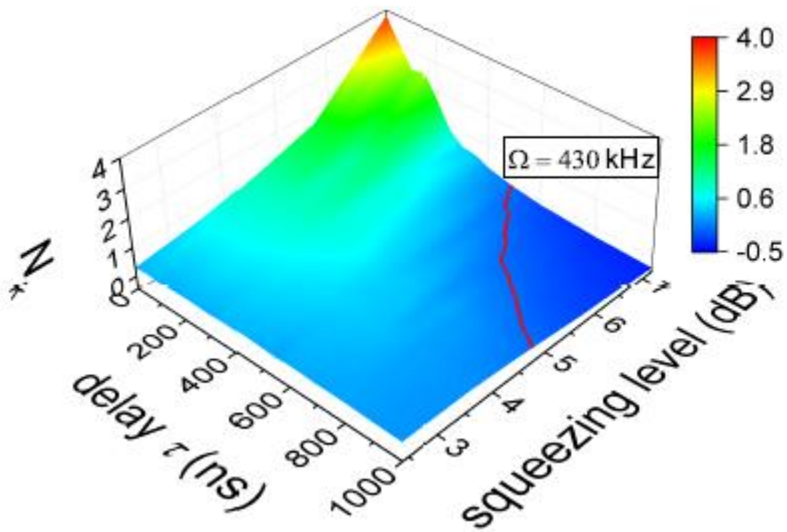


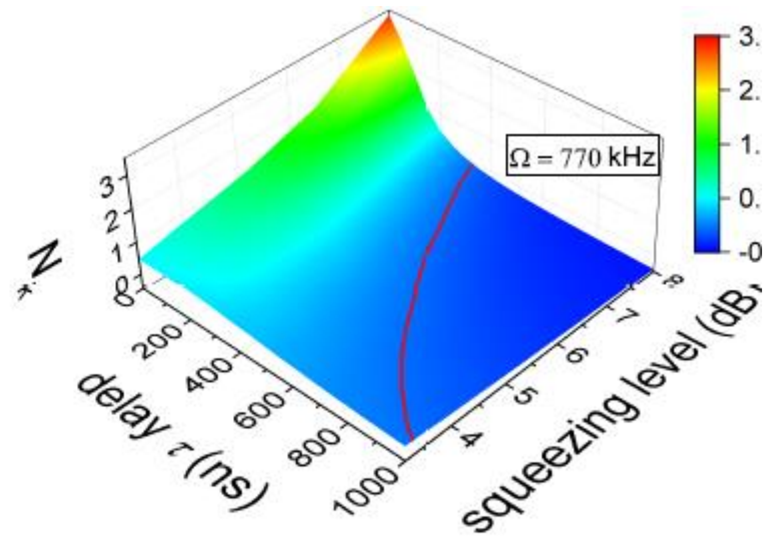
From moments up to 4th-order

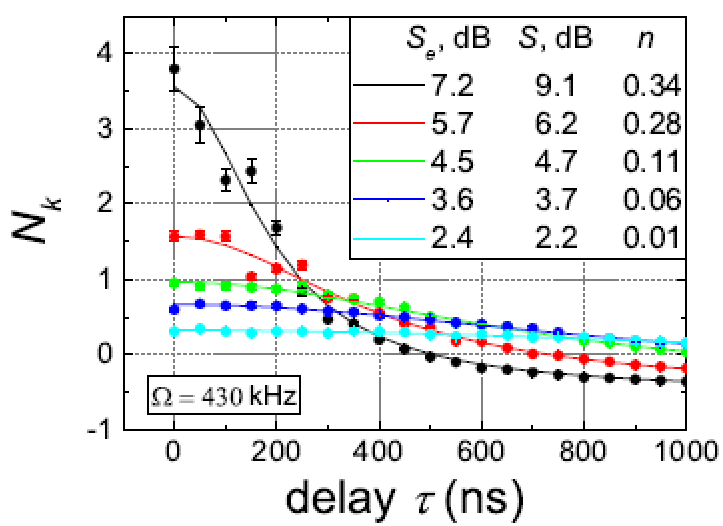
→ Negativity kernel $N_k(\tau)$

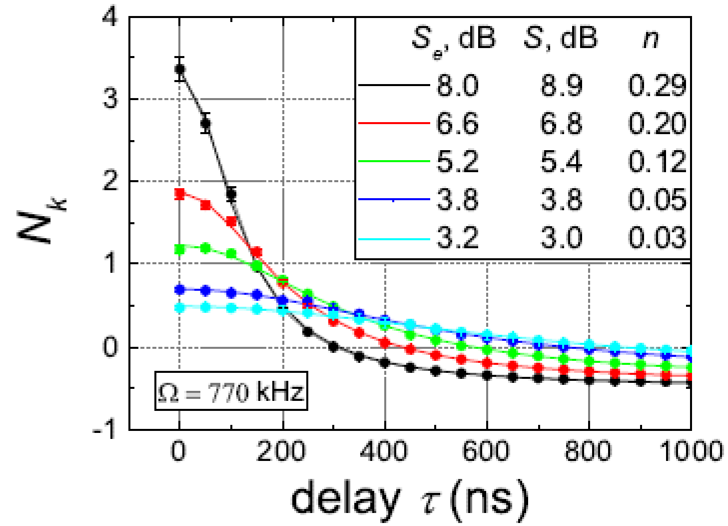
K. G. Fedorov, S. Pogorzalek *et al.*, Sci. Rep. **8**, 6416 (2018).

Finite-time entanglement correlations







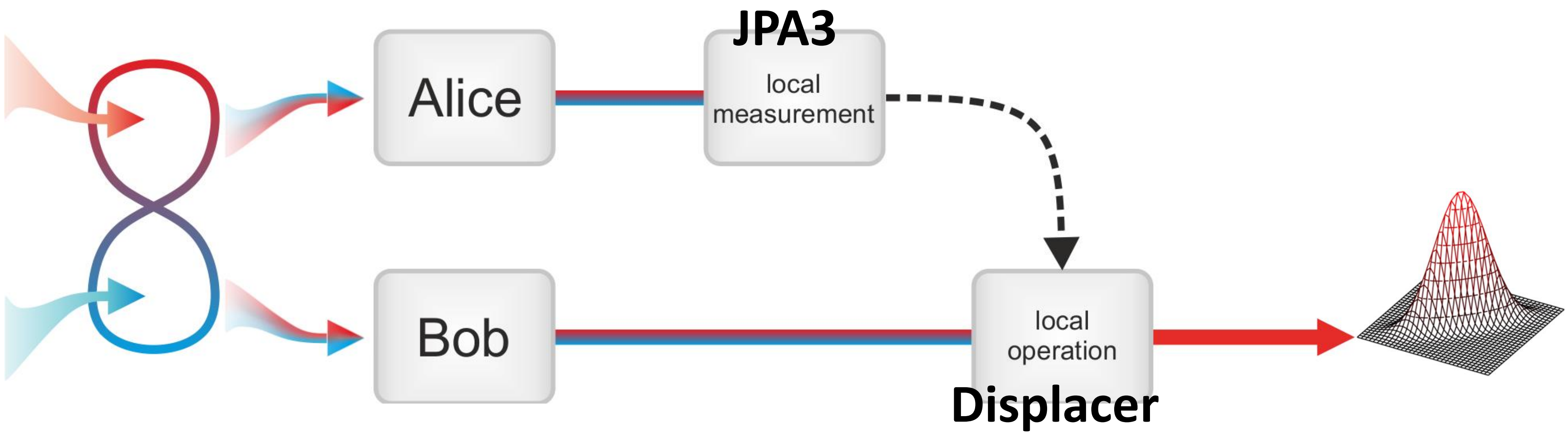


→ Strongly dependent on squeezing level

→ Sufficient for first RSP & QT experiments without delay line

K. G. Fedorov, S. Pogorzalek *et al.*, Sci. Rep. **8**, 6416 (2018).

Remote state preparation



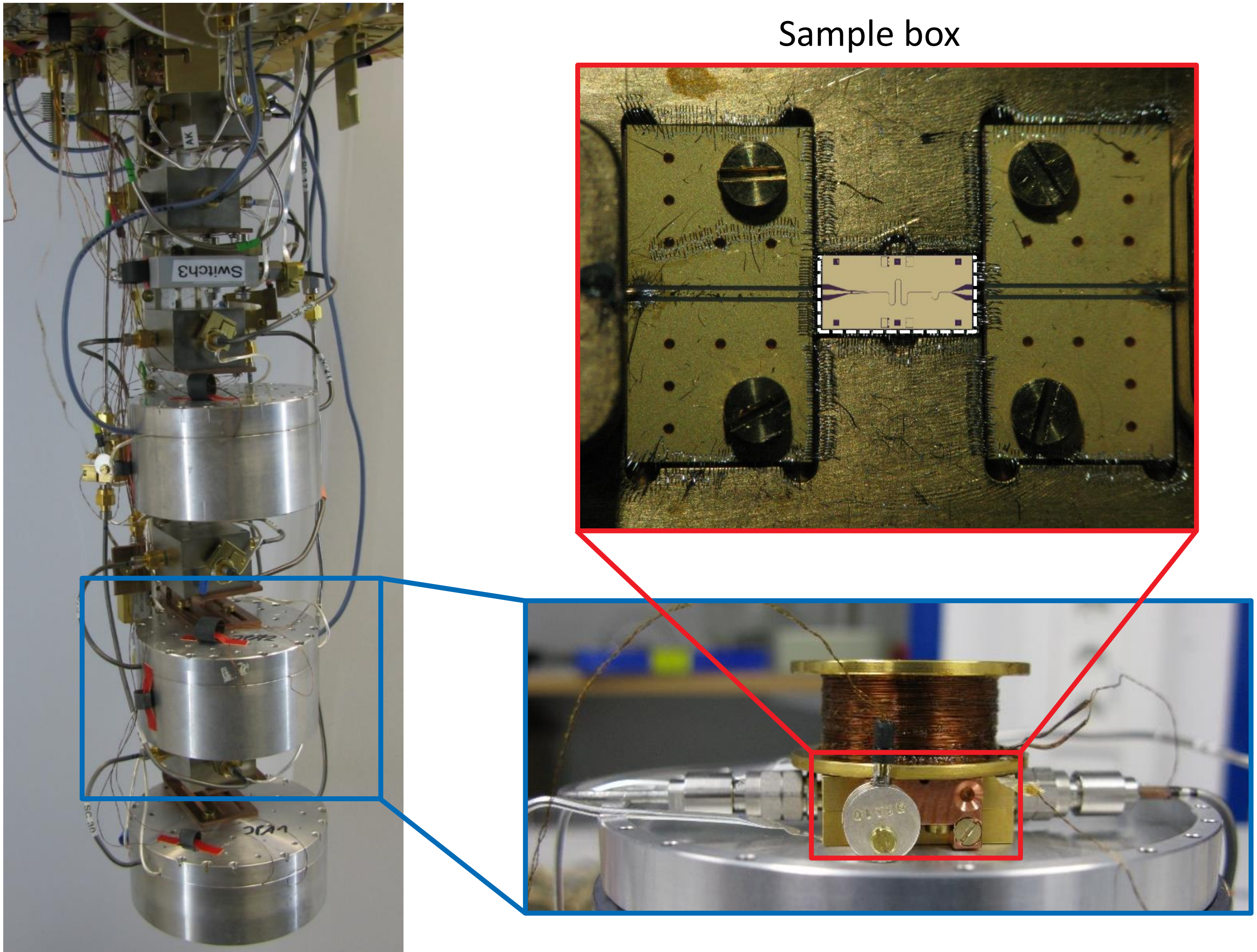
→ Remote state preparation is the variant of quantum state teleportation in which the sender knows the quantum state to be communicated

→ First „real“ communication protocol implemeted with CV propagating microwaves

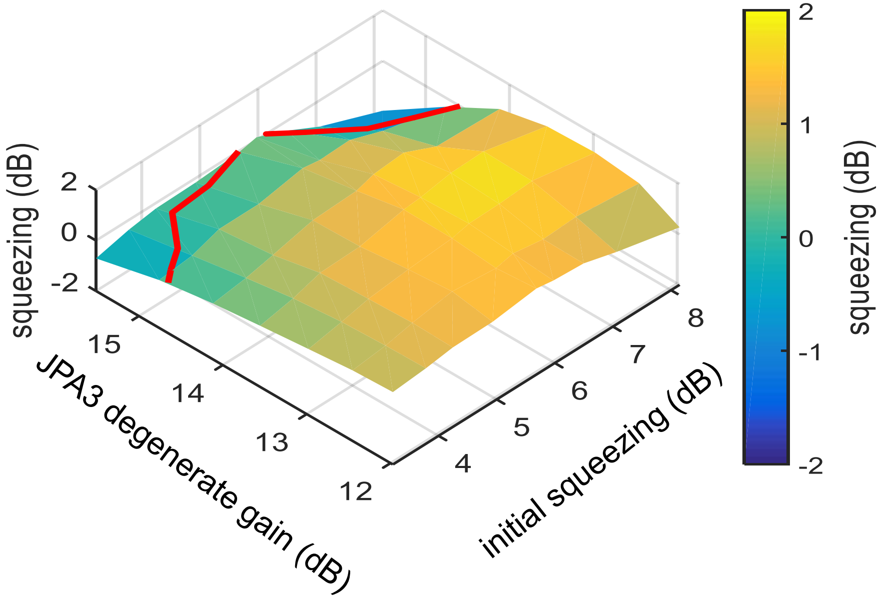
C. H. Bennett *et al.*, Phys. Rev. Lett. **87**, 077902 (2001).

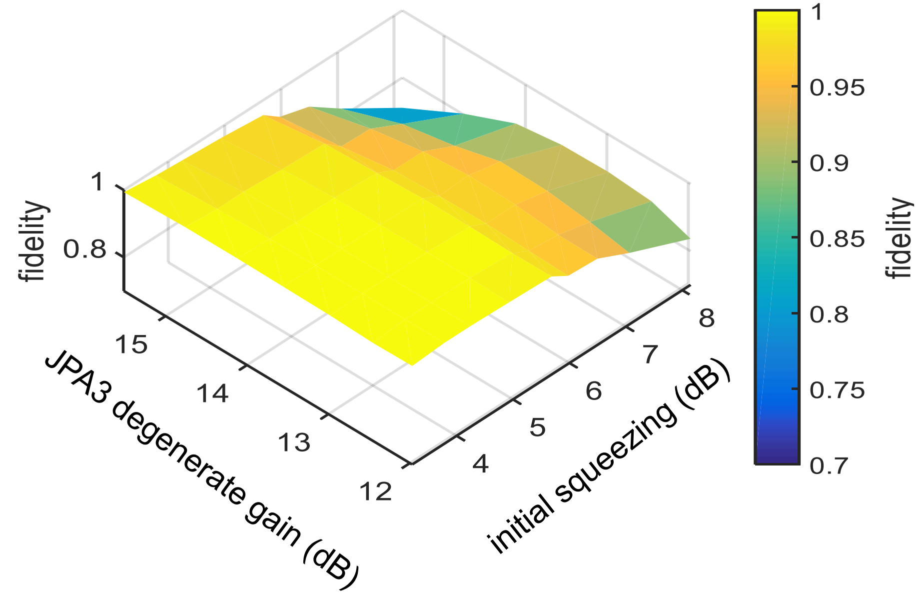
R. Di Candia *et al.*, EPJ Quantum Technology **2**, 25 (2015).
S. Pogorzalez, K. G. Fedorov *et al.*, in preparation.

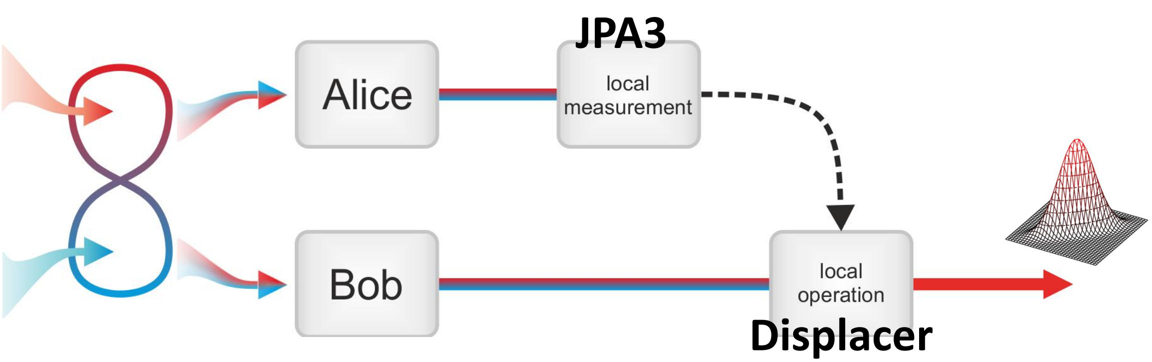
Remote state preparation
Experimental setup



Remote state preparation
Squeezing & Fidelity







→ Maximum fidelity of *F > 0.999* for low initial squeezing

R. Di Candia *et al.*, EPJ Quantum Technology **2**, 25 (2015).
S. Pogorzalez, K. G. Fedorov *et al.*, in preparation.